Central Extensions and Reciprocity Laws for Arithmetic Surfaces

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Numerations in Dimension 1

Example

- $\mathbb{Q}_p \supset \mathbb{Z}_p \longrightarrow \mathbb{Z}_p/p\mathbb{Z}_p = \mathbb{F}_p$
- $\#\mathbb{F}_p = p$
- Assume $\#\mathbb{Z}_p = 1 \Longrightarrow \#p\mathbb{Z}_p = 1/p$
- Assume $\#\mathbb{Z}_p = \alpha \in \mathbb{R}_{>0} \Longrightarrow \#p\mathbb{Z}_p = \alpha/p$
- Numerations for all open compact subgroups
- Similarly, $\mathbb{F}_p((t)) \supset \mathbb{F}_p[[t]]$
- $\log : \mathbb{R}_{>0} \Longrightarrow \mathbb{R}$ Multiplicative$\Longrightarrow$Additive

Numerations in Dimension 1

Definition

- $A, B \leq \mathbb{Q}_p$: open compact
- $[A|B]_1 := \log\left(\#\frac{B}{A\cap B}\right) - \log\left(\#\frac{A}{A\cap B}\right)$
- Systematic Numeration:
  $\mathbf{n} : \{\text{open compact subgroups of } G\} \longrightarrow \mathbb{R}$
  satisfying $\mathbf{n}(B) = \mathbf{n}(A) + [A|B]_1$.
- $\mathrm{Num}(\mathbb{Q}_p) := \{\text{systematic numerations of } \mathbb{Q}_p\}$
- $\mathrm{Num}(\mathbb{Q}_p)$ is an $\mathbb{R}$-torsor
- e.g. $\mathbf{m} = \mathbf{n} + \log \alpha$

Numerations in Dimension 1

Commensurable Subspaces

- G: locally compact group, $A, B \leq G$: open compact
- $[A|B]_1 := \log \# \dfrac{B}{A \cap B} - \log \# \dfrac{A}{A \cap B}$

Definition

- Systematic Numeration:
  $\mathbf{n} : \{\text{open compact subgroups of } G\} \longrightarrow \mathbb{R}$
  satisfying $\mathbf{n}(B) = \mathbf{n}(A) + [A|B]_1$.
- $\mathrm{Num}(G) := \{\text{systematic numerations of } G\}$

Facts

- $\mathrm{Num}(G)$ is an $\mathbb{R}$-torsor

Definition

Example

- $L = \mathbb{Q}_p((u)),\ \ \mathbb{Q}_p\{\{u\}\}$: 2 dimensional local field,
- Ind-Pro Topology
- $\mathbb{Q}_p[[u]]/u\mathbb{Q}_p[[u]] = \mathbb{Q}_p,\ \ \mathbb{Z}_p((u))/p\mathbb{Z}_p((u)) = \mathbb{F}_p((u))$
- Ind-Pro Topology

Definition

- $W_1, W_2 \leq V$: closed subspace
- $W \leq W_1, W \leq W_2$ s.t. W_i/W: locally compact
- $[W_1|W_2; W]_2 := \mathrm{Hom}_{\mathbb{R}}\big(\mathrm{Num}(W_1/W), \mathrm{Num}(W_2/W)\big)$
- $[W_1|W_2]_2 := \mathrm{prolim}_W [W_1|W_2; W]_2$
- $[W_1|W_2]_2$ is an $\mathbb{R}$-torsor

Discrepancy

Discrepancy

- $\overline{V}$: metrized $\mathbb{R}$-space of finite dimension
- $\overline{V}_* : 0 \to \overline{V}_1 \stackrel{i}{\to} \overline{V}_2 \stackrel{\pi}{\to} \overline{V}_3 \to 0$ exact as $\mathbb{R}$-space
- Discrepancy

  $$\gamma(\overline{V}_*) := \frac{\|i(e_1) \wedge \cdots \wedge i(e_r) \wedge e_{r+1} \wedge e_{r+s}\|_2}{\|e_1 \wedge \cdots \wedge e_r\|_1 \cdot \|\pi(e_{r+1}) \wedge \cdots \wedge \pi(e_{r+s})\|_3}$$

Basic Maps: One Example

- $A, B, C \leq \mathbb{R}((u))$ s.t. $A \sim B \sim C$ (Commensurable)
- $(A|B) := \det\left(\frac{A}{A \cap B}\right)^* \otimes \det\left(\frac{B}{A \cap B}\right)$
- $\alpha_{A,B,C} : (A|B) \otimes_{\mathbb{R}} (B|C) \simeq (A|C)$
- $\exists \gamma(\alpha_{\overline{A},\overline{B},\overline{C}}) \in \mathbb{R}$ discrepancy & $\overline{\alpha_{A,B,C}} =: \alpha_{A,B,C} \cdot \gamma(\alpha_{\overline{A},\overline{B},\overline{C}})$
- $\Longrightarrow \quad \overline{\alpha_{A,B,C}} : \overline{(A|B)} \otimes_{\mathbb{R}} \overline{(B|C)} \cong \overline{(A|C)}$ isometry

Central Extension: Finite Places

Definition

- L: 2 dim local field, $\mathcal{O}_L$: valuation ring
- Central Extension:

$$\widehat{L}^* := \{(g, \mathbf{n}) : g \in L^*, \mathbf{n} \in [\mathcal{O}_L | g\mathcal{O}_L]_2\}$$

$$(g_1, \mathbf{n}_1)(g_2, \mathbf{n}_2) := (g_1 g_2, \mathbf{n}_1 \cdot g_1(\mathbf{n}_2))$$

$$\begin{aligned} \text{w/} \quad \mathbf{n}_1 \otimes g_1(\mathbf{n}_2) \in & [\mathcal{O}_L | g_1\mathcal{O}_L]_2 \otimes_R g_1\left([\mathcal{O}_L | g_2\mathcal{O}_L]_2\right) \\ \simeq & [\mathcal{O}_L | g_1\mathcal{O}_L]_2 \otimes_{\mathbb{R}} [g_1\mathcal{O}_L | g_1 g_2\mathcal{O}_L]_2 \\ = & [\mathcal{O}_L | g_1 g_2\mathcal{O}_L]_2 \end{aligned}$$

Fact

- $\exists$ exact sequence

$$0 \to \mathbb{R} \to \widehat{L}^* \xrightarrow{\pi} L^* \to 1$$

Symbol (I)

- $f', g' \in \widehat{L}^*$ s.t. $\pi(f') = f$, $\pi(g') = g$
- Definition:

$$\langle f, g\rangle_L := [f', g'] \in \mathbb{R} \subset \widehat{L}^* \quad \forall f, g \in L^*$$

Symbol (I')

- $$\nu_L : L^* \times L^* \longrightarrow K_2(L) \xrightarrow{\partial_2} \overline{L}^* \xrightarrow{\partial_1} \mathbb{Z}$$

  w/ ∂_2: tame symbol, ∂_1: valuation

Theorem

$$\langle f, g\rangle_L = \log \#(\overline{L}) \cdot \nu_L(f, g)$$

Symbols at Infinity

Central Extension

- $L = K((t)) \supset K[[t]]$ w/ $K = \mathbb{R}$, or $\mathbb{C}$
- Central Extension:
  $$\widehat{L^*}^{\mathrm{ar}} := \{(g,a) : g \in L^*, a \in \overline{(K[[t]]|gK[[t]])}, a \neq 0\}$$
  $$\text{w/ } (g_1,a_1)(g_2,a_2) := (g_1g_2, a_1g_1(a_2))$$
- $\exists$ exact sequence
  $$0 \to \mathbb{R}^* \to \widehat{L^*}^{\mathrm{ar}} \xrightarrow{\pi} L^* \to 1$$

Symbol(II)

- $f, g \in L^*$, $f', g' \in \widehat{L^*}^{\mathrm{ar}}$ s.t. $\pi(f') = f$, $\pi(g') = g$
- Local Symbol: $\langle f, g\rangle_L := [f', g'] \in \mathbb{R}$
- Fact: $$\langle f, g\rangle_L = \log \frac{|f_0(0)^{\nu_t(g)}|}{|g_0(0)^{\nu_t(f)}|}$$
  $$\text{w/ } f(t) = t^{\nu_t(f)} f_0(t), \quad g(t) = t^{\nu_t(g)} g_0(t)$$

Preparations

Setting

- $X/\mathcal{O}_F$: arithmetic surface w/ $\pi : X \to \operatorname{Spec} \mathcal{O}_F$
- $C \subset X$: integral curve, η_C: generic pt , $x \in C$: closed pt

Finite Places

- $k(X)_{x,C} := \mathrm{Frac}\Big(\widehat{\big(\widehat{\mathcal{O}}_{X,x}\big)_{\eta_C}}\Big) = \bigoplus_{2 \text{ dim loc fd}} L$
- $\langle \cdot, \cdot \rangle_{x,C} := \bigoplus \langle \cdot, \cdot \rangle_L$

Infinite Places

- $P \in X_F$: closed point
- $k(X)_P \widehat{\otimes}_{\mathbb{Q}} \mathbb{R} := \mathrm{Frac}\Big(\widehat{\mathcal{O}_{X_F,P} \otimes_{\mathbb{Q}} \mathbb{R}}\Big) = \bigoplus_{L = K((t))\, K = \mathbb{R}, \text{or } \mathbb{C}} L$
- $\langle \cdot, \cdot \rangle_P := \bigoplus \langle \cdot, \cdot \rangle_L$

Reciprocity Law

Reciprocity Law: Around A Point

- $x \in X$: closed point

$$\sum_{C:C\ni x} \langle f, g \rangle_{x,C} = 0$$

Reciprocity Law: Along Vertical Curve

- $V \subset X$: vertical curve

$$\sum_{x:x\in V} \langle f, g \rangle_{x,V} = 0$$

Reciprocity Law: Along Horizontal Curve

- $P \in X_F$: closed pt, E_P: corr. horizontal curve

$$\sum_{x:x\in E_P} \langle f, g \rangle_{x,E_P} + \langle f, g \rangle_P = 0$$

Central Extension: Global

Central Extension

- X: arithmetic surface
- $\exists$ central extension

$$0 \to \mathbb{R} \to \widehat{k(X)^*}_{W^{\mathrm{ar}}_{\bar{O}_X}} \to k(X)^* \to 1$$

- $W^{\mathrm{ar}}_{\bar{O}_X} := \prod_{x \in E_P} \mathcal{O}_{E_P,x}((u)) \times \left(\overline{\mathcal{O}_{X_F}}\big|_P((u))\widehat{\otimes}_{\mathbb{Q}}\mathbb{R}\right)$

Main Theorem

Exact sequence

$$0 \to \mathbb{R} \to \widehat{k(X)^*}_{W^{\mathrm{ar}}_{\bar{O}_X}} \to k(X)^* \to 1$$

splits

General Construction

w/o Adjunction Formula

- $\omega := \frac{\sqrt{-1}}{2g} \sum_{i=1}^{g} \omega_i \wedge \overline{\omega}_i$: canonical volume form/$X_\infty$
- $\overline{\mathcal{L}}$: ω-admissible metrized line bdl/X
- $s \neq 0$: rat section of $\mathcal{L}$ s. t. $s|_{E_P} \not\equiv 0$
- $\mathrm{div}(s|_{E_P}) =: \sum_x n_x[x]$, $\mathrm{div}(s|_{X_F}) =: \sum_Q m_Q[Q]$
- $\exists c(s) \in \mathbb{R}$ s.t.

$$\overline{\mathcal{L}} = \overline{\mathcal{O}(\mathrm{div}(s))} \cdot \exp(-c(s))$$

- $$W^{\mathrm{ar}}_{\overline{\mathcal{L}},s} := \prod_{x \in E_P} \mathfrak{m}_{E_P,x}^{-n_x}((u)) \times \prod_{Q \in X_F} \overline{\mathcal{O}_{X_F}(Q)}^{\otimes m_Q}\Big|_P((u)) \,\widehat{\otimes}_{\mathbb{Q}}\mathbb{R} \cdot \mathbf{e}^{-c(s)}$$

Numeration: Arith Intersection

Definition

- $\mathrm{num}_0\big(\mathfrak{m}_{E_P,x}\big) := -\log q_x$;
- $\mathrm{num}_0\big(\overline{\mathcal{O}_{X_F}(Q)|_P}\big) := g(Q,P)$; Arakelov-Green function
- $\mathrm{num}_0\big(\mathcal{O}_{E_P,x}[[u]]/u^n\mathcal{O}_{E_P,x}[[u]]\big) := n$;
- $\mathrm{num}_0\big(\mathcal{O}_{X_F,P}[[u]]/u^m\mathcal{O}_{X_F,P}[[u]]\big) := m$.

Facts

- $\mathrm{num}_0^{\mathrm{coef}}\big(W^{\mathrm{ar}}_{\overline{\mathcal{L}},s}\big) = \deg_{\mathrm{Ar}}\big(\overline{\mathcal{L}}|_{\overline{E_P}}\big)$
- s_1 and s_1', resp. s and s': 'nice' sections of $\mathcal{L}_1$, resp. of $\mathcal{L}$
- $\Longrightarrow$ $\exists$ metrized $\mathbb{R}$-torsors isometries

$$\mathrm{Num}\big(W^{\mathrm{ar}}_{\overline{\mathcal{L}},s}\big) \cong \mathrm{Num}\big(W^{\mathrm{ar}}_{\overline{\mathcal{L}},s'}\big),$$

$$\mathrm{Num}\big(W^{\mathrm{ar}}_{\overline{\mathcal{L}}_1,s_1}/W^{\mathrm{ar}}_{\overline{\mathcal{L}},s}\big) \cong \mathrm{Num}\big(W^{\mathrm{ar}}_{\overline{\mathcal{L}}_1,s_1'}/W^{\mathrm{ar}}_{\overline{\mathcal{L}},s'}\big).$$

Global Numeration

Definition

$$[W^{\mathrm{ar}}_{\overline{\mathcal{L}}_1,s_1} | W^{\mathrm{ar}}_{\overline{\mathcal{L}}_2,s_2}]_2$$
$$:=\mathrm{prolim}_{(\overline{\mathcal{L}},s)}\mathrm{Hom}_{\mathbb{R}}\Big(\mathrm{Num}\big(W^{\mathrm{ar}}_{\overline{\mathcal{L}}_1,s_1}/W^{\mathrm{ar}}_{\overline{\mathcal{L}},s}\big),\mathrm{Num}\big(W^{\mathrm{ar}}_{\overline{\mathcal{L}}_2,s_2}/W^{\mathrm{ar}}_{\overline{\mathcal{L}},s}\big)\Big)$$

Proposition

$$[W^{\mathrm{ar}}_{\overline{\mathcal{L}}_1,s_1} | W^{\mathrm{ar}}_{\overline{\mathcal{L}}_2,s_2}]_2 \cong [W^{\mathrm{ar}}_{\overline{\mathcal{L}}_1,s_1'} | W^{\mathrm{ar}}_{\overline{\mathcal{L}}_2,s_2'}]_2$$

Action on Reference Spaces

Examples

- $f \in k(X)^*$ s.t. $f|_{E_P} \not\equiv 0$,
  $\mathrm{div}(f|_{E_P}) =: \sum_x n_x[x]$, $\mathrm{div}(f|_{X_F}) =: \sum_Q m_Q[Q] \Longrightarrow$
- Algebraic:
  $$f \cdot W^{\mathrm{ar}}_{\mathcal{O}_X} = \prod_{x \in E_P} \mathfrak{m}_{E_P,x}^{-n_x}((u)) \times \prod_{Q \in X_F} \mathcal{O}_{X_F}(Q)^{m_Q}|_P((u))$$
  w/ $W^{\mathrm{ar}}_{\mathcal{O}_X} = \prod_{x \in E_P} \mathcal{O}_{E_P,x}((u)) \times \prod_{Q \in X_F} \mathcal{O}_{X_F,Q}|_P((u))$.
- Metric:
  $$f \cdot W^{\mathrm{ar}}_{\overline{\mathcal{O}}_X} := \prod_{x \in E_P} \mathfrak{m}_{E_P,x}^{-n_x}((u)) \times \prod_{Q \in X_F} \overline{\mathcal{O}_{X_F}(Q)}^{\otimes m_Q}|_P((u)) \cdot e^{\int_{X_\infty} -\log\|f\|d}$$
  $$= W^{\mathrm{ar}}_{\overline{\mathcal{O}}_X(\mathrm{div}_{\mathrm{ar}}(f)),f} .$$
  w/ $\mathrm{div}_{\mathrm{ar}}(f)$: Arakelov divisor of f
- $$\boxed{f \cdot W^{\mathrm{ar}}_{\overline{\mathcal{O}}_X,1} := W^{\mathrm{ar}}_{\overline{\mathcal{O}}_X(\mathrm{div}_{\mathrm{ar}}(f)),f} \quad \forall f \in k(X)^*}$$

Arithmetic Central Extension

Definition

- Define an arithmetic central extension by:
  - $\widehat{k(X)^*}_{W^{\rm ar}_{\mathcal{O}_X}} := \{(f,\alpha) : f \in k(X)^*, \alpha \in [W^{\rm ar}_{\mathcal{O}_X}|fW^{\rm ar}_{\mathcal{O}_X}]_2\};$
  - $(f,\alpha)\circ(g,\beta) := (fg, \alpha\circ f(\beta)) \qquad$ w/ $\alpha\circ f(\beta) := \alpha\otimes f(\beta).$
- Based on:

$$[W^{\rm ar}_{\mathcal{O}_X}|fW^{\rm ar}_{\mathcal{O}_X}]_2 \otimes [fW^{\rm ar}_{\mathcal{O}_X}|fgW^{\rm ar}_{\mathcal{O}_X}]_2 \cong [W^{\rm ar}_{\mathcal{O}_X}|fgW^{\rm ar}_{\mathcal{O}_X}]_2$$

Main Theorem

- $\widehat{k(X)^*}_{W^{\rm ar}_{\mathcal{O}_X}}$ is a central extension of $k(X)^*$ by $\mathbb{R}$;
- Canonical exact sequence splits

$$0 \to \mathbb{R} \longrightarrow \widehat{k(X)^*}_{W^{\rm ar}_{\mathcal{O}_X}} \longrightarrow k(X)^* \to 1$$

Algebraic Residue Theory

Residue Isomorphism

- $E_P =: \mathrm{Spec}\, A$, w/ A: order of $\mathcal{O}_F$, a Dedekind domain
- $W_{E_P/Y} := \{b \in F(P) : \mathrm{Tr}(bA) \subset \mathcal{O}_F\}$.
- $W_{E_P/Y} := \prod_{x \in E_P} \mathfrak{m}_{E_P,x}^{b_x}$
- Residue map induces

$$\mathrm{res} : \mathcal{K}_\pi(E_P)|_{E_P} \simeq \widetilde{W_{E_P/Y}},$$

Arith Adjunction Formula

- Set: $\mathbf{d}_\lambda(\overline{E}_P) := \sum_{\sigma \in S_\infty} N_\sigma \mathbf{d}_{g,\sigma}(\overline{E}_P)$
  w/ $\mathbf{d}_{g,\sigma}(\overline{E}_P) := \sum_{i \leq j} g(P_i, P_j)$ & $P = \{P_i\}$
- Introduce: $\mathbf{d}_{E_P/Y} := -\log\left(W_{E_P/Y} : \mathcal{O}_F\right) \Longrightarrow$
- Arith Adjunction Formula: $\overline{\mathcal{K}}_\pi \cdot \overline{E}_P + \overline{E}_P^2 = \mathbf{d}_{E_P/Y} + \mathbf{d}_\lambda(\overline{E}_P)$

Arithmetic Adelic Complex

Definition

- $P \in X_F$: alg pt, $E_P \subset X$: corr horizontal curve
  $\mathcal{I}_{\mathcal{E}_\mathcal{P}}$: ideal sheaf of $E_P \subset X$
- $\mathcal{L}$: metrized line bdl/X, $\mathcal{L}|_{E_P} := \mathcal{O}_{E_P}(\sum_x n_x[x])$
  $\mathcal{L}_F := \mathcal{L}|_{X_F} := \mathcal{O}_{X_F}(\sum_{Q\in X_F} m_Q[Q])$
- Introduce adelic complex $\mathcal{A}^{\mathrm{Ar}}_{\overline{E}_P,*}(\overline{\mathcal{L}})$:

$$\widehat{k(X)}_{E_P} \times \Big(\prod_{x\in E_P} (B_x \otimes_{\widehat{\mathcal{O}}_{X,x}} \mathcal{L}) \times \big(B_P \otimes_{\widehat{\mathcal{O}}_{X,x}} \overline{\mathcal{L}}\, \widehat{\otimes}_{\mathbb{Q}} \mathbb{R}\big)\Big) \longrightarrow \mathbb{A}^{\mathrm{Ar}}_{\overline{E}_P}$$

w/ $B_x := \mathcal{O}_{E_P,x}((u)),\ B_P = \mathcal{O}_{X_F}\big|_P((u))$,

$$\mathbb{A}^{\mathrm{Ar}}_{\overline{E}_P} := \prod\nolimits'_{x\in \overline{E}_P} \widehat{k(E_P)}_x((u))$$

Arithmetic Cohomology

Proposition

$$\begin{aligned} H^0\left(\mathcal{A}^{\mathrm{Ar}}_{\overline{E}_P,*}(\overline{\mathcal{L}})\right) &= H^0_{\mathrm{ar}}\left(\overline{E}_P, \overline{\mathcal{L}}|_{\overline{E}_P}\right)((u)), \\ H^1\left(\mathcal{A}^{\mathrm{Ar}}_{\overline{E}_P,*}(\overline{\mathcal{L}})\right) &= H^1_{\mathrm{ar}}\left(\overline{E}_P, \overline{\mathcal{L}}|_{\overline{E}_P}\right)((u)). \end{aligned}$$

Coefficient Numeration

$$\begin{aligned} \mathrm{num}_0\left(H^0_{\mathrm{ar}}\left(\overline{E}_P, \overline{\mathcal{L}}|_{\overline{E}_P}\right)\right) &= h^0_{\mathrm{ar}}\left(\overline{E}_P, \overline{\mathcal{L}}|_{\overline{E}_P}\right) \\ &:= -\log \sum_{x\in H^0_{\mathrm{ar}}\left(\overline{E}_P, \overline{\mathcal{L}}|_{\overline{E}_P}\right)} e^{-\pi\|x\|} \\ \mathrm{num}_0\left(H^1_{\mathrm{ar}}\left(\overline{E}_P, \overline{\mathcal{L}}|_{\overline{E}_P}\right)\right) &= h^1_{\mathrm{ar}}\left(\overline{E}_P, \overline{\mathcal{L}}|_{\overline{E}_P}\right) \\ &= -\log \sum_{x\in H^0_{\mathrm{ar}}\left(\overline{E}_P, K_{\overline{E}_P}\otimes\overline{\mathcal{L}}|_{\overline{E}_P}\right)} e^{-\pi\|x\|} \end{aligned}$$

Numeration in Dimension 2

Numeration in Dimension 2

- Coef Numeration + $((u))$ numeration:
  $\mathrm{num}_0\big(A[[u]]\big/u^nA[[u]] := n\cdot\mathrm{num}_0(A)$
  w/ A: numerable locally compact space
  $\Longrightarrow \mathbb{R}$-torsor $\mathrm{Num}(H^i_{\mathrm{Ar}}(\overline{E}_P, \overline{\mathcal{L}}|_{\overline{E}_P})((u))$
- $\Longrightarrow \mathbb{R}$-torsor $\mathrm{Num}(H^i(\mathcal{A}^{\mathrm{Ar}}_{\overline{E}_{P,*}}(\overline{\mathcal{L}})))$

Numeration for Adelic Arithmetic Complex

- Definition:

$$\mathrm{Num}(\mathcal{A}^{\mathrm{Ar}}_{\overline{E}_{P,*}}(\overline{\mathcal{L}}))$$
$$:= \mathrm{Hom}_{\mathbb{R}}\big(\mathrm{Num}(H^1(\mathcal{A}^{\mathrm{Ar}}_{\overline{E}_{P,*}}(\overline{\mathcal{L}}))), \mathrm{Num}(H^0(\mathcal{A}^{\mathrm{Ar}}_{\overline{E}_{P,*}}(\overline{\mathcal{L}})))\big)$$

Main Theorem

Main Theorem

∃ canonical isometry

$$[W^{\mathrm{ar}}_{\overline{\mathcal{L}}_1,s_1} \,|\, W^{\mathrm{ar}}_{\overline{\mathcal{L}}_2,s_2}]_2 \cong \mathrm{Hom}_{\mathbb{R}}\Big(\mathrm{Num}(\mathcal{A}^{\mathrm{Ar}}_{\overline{E}_P,*}(\overline{\mathcal{L}}_1)), \mathrm{Num}(\mathcal{A}^{\mathrm{Ar}}_{\overline{E}_P,*}(\overline{\mathcal{L}}_2))\Big)$$

History

History

- Weil: reciprocity law for curves
- Tate: residue formula using trace for curves
- Arbarello-De Concini-Kac: central extension and reciprocity law for curves
- Parshin: reciprocity laws for geometric surfaces
- Osipov: dimension theory in dimension 2 and central extensions for geometric surfaces
- Sugahara-Weng: reciprocity laws and central extensions for arithmetic surfaces

Joint work

- This is a joint work with K. Sugahara

Thank You

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