$$\Rightarrow d\mu \rightsquigarrow \frac{2^{r_1+r_2-1}}{\sqrt{r_1+r_2}} \cdot \underset{\det(\log\|\varepsilon_j^{(i)}\|)}{\underset{\shortparallel}{R}} \; \frac{dY_0}{Y_0}\, dY_1 \cdots dY_{r_1+r_2-1} \cdot \prod_{\sigma:\mathbb{R}} dx_\sigma \cdot \prod_{\tau:\mathbb{C}} dz_\tau =: d\omega$$

Lem
(i) (Boundary of ∂D_T in Y's).

<u>two parts</u>: $\left\{\begin{array}{l}\text{part of the bdy of } \mathcal{D}_{SL O_F\oplus\mathfrak{a}}, \quad \underline{\text{in pair}} \\ \text{hyperplane of } \mathcal{D}_T \text{ defined by the distance } d(\eta, z_\lambda) = \frac{1}{T}\end{array}\right.$

<u>becomes</u>: $Y_0 = T' = N(\mathfrak{a}\mathfrak{b}^{-2}) \cdot T$.

(ii) $d\omega\big|_{Y_0=T'} (d\mu_\Sigma) \quad \frac{\sqrt{r_1+r_2}}{T'} \cdot 2^{r_1+r_2-1} \cdot R\, dY_1 \cdots dY_{r_1+r_2-1} \prod_{\sigma:\mathbb{R}} dx_\sigma \cdot \prod_{\tau:\mathbb{C}} dz_\tau.$

(iii) if ν is the unit normal to the bdy hyperplane of distance

$\nu = \left(\frac{1}{\sqrt{r_1+r_2}\,T'}, 0, \cdots, 0\right)$ & $\frac{\partial f}{\partial \nu} = \left(\frac{1}{\sqrt{r_1+r_2}\,T'}, \cdots 0\right)$ grad f
$= \sqrt{r_1+r_2} \cdot T' \cdot \frac{\partial f}{\partial Y_0}$

since $\left\langle \frac{\partial}{\partial Y_0}, \frac{\partial}{\partial Y_0} \right\rangle = g''\big|_{Y_0=T'} = (r_1+r_2)T')^2$.

$$\Rightarrow \iint_{\mathcal{D}_T} \hat{E}_{\mathfrak{a}}(z,s)\, d\mu = \underbrace{\frac{1}{r_1+4r_2} \cdot \frac{1}{s(s-1)} \sum_{i=1}^{h}}_{\text{others are in pair opposite direction}} \iint_{X_i(T)} \frac{\partial \hat{E}_{\mathfrak{a}}(z,s)}{\partial \nu}\, ds$$

$Y_0 = T_i'$. $i=1,\cdots,h$.

$\iint_{X_i(T)} \frac{\partial}{\partial \nu}(A_{0i} Y_0^s + B_{0i} Y_0^{1-s})\, d\mu$

$$= \frac{r_1+r_2}{r_1+4r_2} \cdot 2^{r_1+r_2-1} \cdot R \cdot |D_F|^{\frac{1}{2}} \cdot \sum_{i=1}^{h} N(\mathfrak{a}\mathfrak{b}_i^{-2}) \cdot \left(\frac{A_{0i}}{s-1} (T_i')^{s-1} - \frac{B_{0i}}{s} (T_i')^{-s} \right)$$

<u>Thm</u> (Weng) Up to a constant factor depending on F, we have

$$\iint_{\mathcal{D}_T} \hat{E}_{\mathfrak{a}}(z,s)\, d\mu = \frac{\hat{\zeta}_F(s)}{s-1} \cdot T^{s-1} - \frac{\hat{\zeta}_F(2-s)}{s} T^{-s}.$$