Set $Y_0 := N(y_{\sigma_1},\dots,y_{\sigma_{r_1}}; v_{\tau_1},\dots,v_{\tau_{r_2}}) = \prod_{\sigma\in\mathbb{R}} y_\sigma \cdot \prod_{\tau\in\mathbb{C}} v_\tau^2$

$$Y_1 := \frac{1}{2}\left(\sum_{i=1}^{r_1} e_i^{(1)} \log y_{\sigma_i} + \sum_{j=1}^{r_2} e_{r_1+j}^{(1)} \log v_{\tau_j}^2\right)$$

$$\vdots$$

$$Y_{r_1+r_2-1} := \frac{1}{2}\left(\sum_{i=1}^{r_1} e_i^{(r_1+r_2-1)} \log y_{\sigma_i} + \sum_{j=1}^{r_2} e_{r_1+j}^{(r_1+r_2-1)} \log v_{\tau_j}^2\right)$$

$$\Rightarrow \quad y_{\sigma_i} = Y_0^{\frac{1}{r_1+r_2}} \prod_{q=1}^{r_1+r_2-1} |\varepsilon_q^{(i)}|^{2Y_q} \quad (i=1,\dots,r_1)$$

$$v_{\tau_j}^2 = Y_0^{\frac{1}{r_1+r_2}} \prod_{q=1}^{r_1+r_2-1} \left(|\varepsilon_q^{(r_1+j)}|^{2Y_q}\right)^2 .$$

$$\Rightarrow \quad \mathcal{D}_{\Gamma_q} \longleftrightarrow \begin{cases} 0 < Y_0 < \infty \\ -\frac{1}{2} \le Y_1,\dots,Y_{r_1+r_2-1} \le \frac{1}{2} \\ (x_{\sigma_1},\dots,x_{\sigma_{r_1}}; z_{\tau_1},\dots,z_{\tau_{r_2}}) \in \mathcal{F}_q(\mathfrak{a}\mathfrak{d}^{-1}) \end{cases}$$

$$\simeq \mathcal{D}_{\mathfrak{a}\mathfrak{d}^{-1}} \subset \mathbb{R}^{r_1}\times\mathbb{C}^{r_2}$$

$$\Rightarrow \quad \frac{\partial y_{\sigma_i}}{\partial Y_0} = \frac{1}{(r_1+r_2)Y_0} y_{\sigma_i}, \quad i=1,\dots,r_1$$

$$\frac{\partial y_{\sigma_i}}{\partial Y_q} = 2\cdot\log|\varepsilon_q^{(i)}|\cdot y_{\sigma_i}, \quad i=1,\dots,r_1, \quad q=1,\dots,r_1+r_2-1$$

$$\frac{\partial t_{\tau_j}}{\partial Y_0} = \frac{1}{(r_1+r_2)Y_0} t_{\tau_j}, \quad j=1,\dots,r_2$$

$$\frac{\partial t_{\tau_j}}{\partial Y_q} = 2\log|\varepsilon_q^{(j)}|^2 t_{\tau_j}, \quad j=1,\dots,r_2, \quad q=1,\dots,r_1+r_2-1 .$$

$$\Rightarrow \quad g = \begin{pmatrix} g_{\mathrm{Im} J} & 0 \\ 0 & g_{\mathrm{Re} Z} \end{pmatrix} \text{ w/ } g_{\mathrm{Im} J} = g_{\mathrm{Re} Z} = \begin{pmatrix} \frac{1}{y_1^2} & & & & & \\ & \ddots & & & 0 & \\ & & \frac{1}{y_{r_1}^2} & & & \\ & & & \left(\frac{1}{v_1^2}\right)^2 & & \\ & 0 & & & \ddots & \\ & & & & & \left(\frac{1}{v_{r_2}^2}\right)^2 \end{pmatrix}$$

$$\tilde{g}_{ij} = \sum_{\alpha\beta} \frac{\partial x^\alpha}{\partial \tilde{x}^i}\cdot\frac{\partial x^\beta}{\partial \tilde{x}^j}\, g_{\alpha\beta}$$