

### I.3. Rank Two $\mathcal{O}_F$ -lattices: Upper half Space Model

- $\Lambda = (P, \mathcal{B})$  rank two  $\mathcal{O}_F$ -lattice  $\Rightarrow P = \mathcal{O}_F \oplus \mathcal{O}_2 \hookrightarrow F^2 \hookrightarrow (\mathbb{R}^{r_1} \times \mathbb{C}^{r_2})^2$
- $\mathcal{B}$ : parametrized by  $\mathbb{H}^{r_1} \times \mathbb{H}^{r_2} \ni (z, w) \in S_{2n}$  w/  $\begin{cases} z = iz = (0, 1) \\ w = jw = (0, 1) \end{cases}$   $\begin{matrix} \text{IR} \\ \text{C} \end{matrix}$   $(\mathbb{R}^{r_1})^{r_1} \times (\mathbb{C}^{r_2})^{r_2}$
- Isometry leads to  $SL(\mathcal{O}_F \oplus \mathcal{O}_2) = GL_2(F) \backslash \left( \begin{matrix} \text{OR } a \\ a^* \text{OR } \end{matrix} \right)$   $\mathcal{B} = (\mathcal{B}_\sigma; \mathcal{B}_\tau)$    
  $\uparrow$  real  $\uparrow$  complex

$\Rightarrow \tilde{A}(\mathcal{P}) = SL(\mathcal{O}_F \oplus \mathcal{O}_2) \backslash \mathbb{H}^{r_1} \times \mathbb{H}^{r_2} \cong$  moduli space of rank 2  $\mathcal{O}_F$ -modules of volume  $N_{F/\mathbb{Q}} \cdot |\Delta_F|$ .

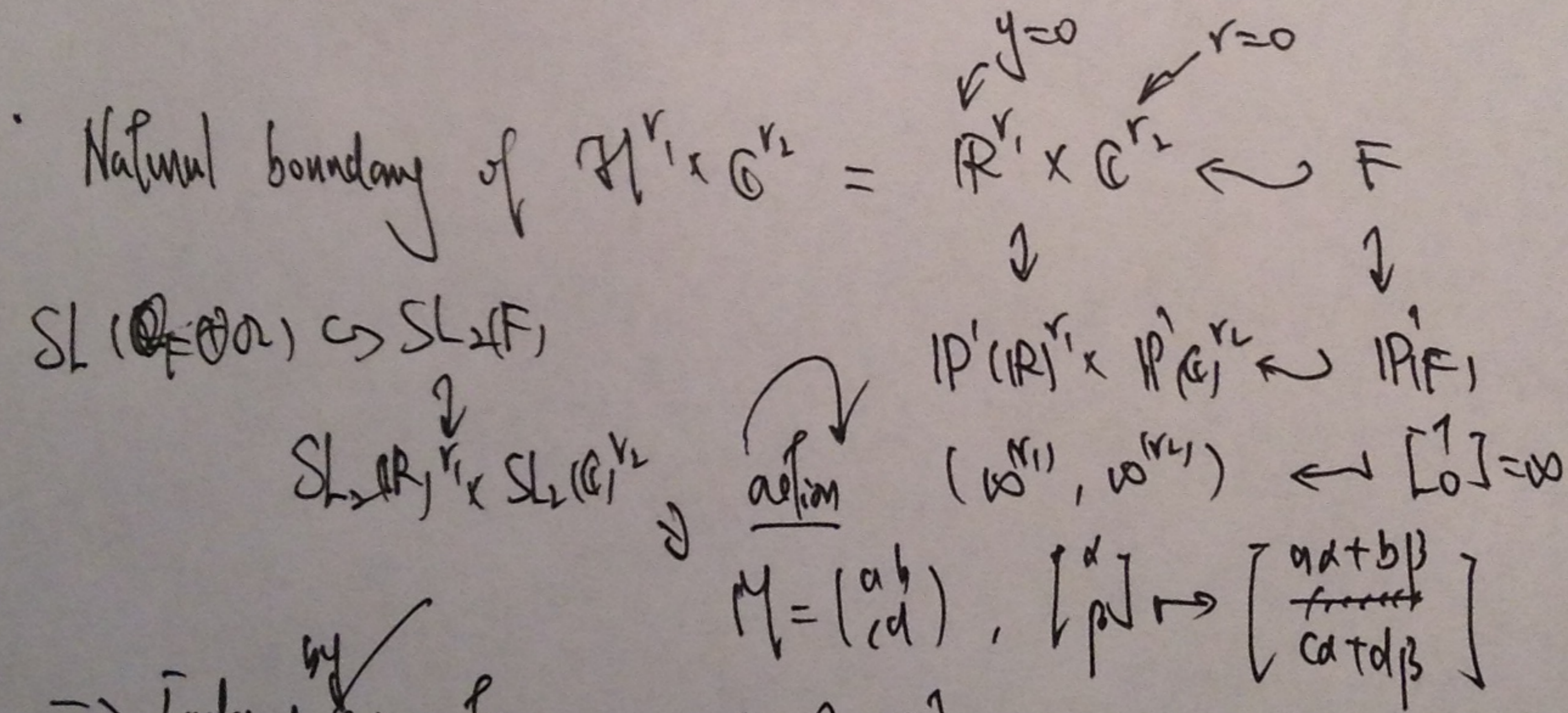
$M_{F^2} [N_{F/\mathbb{Q}} \cdot |\Delta_F|] \cong (SL(\mathcal{O}_F \oplus \mathcal{O}_2) \backslash \mathbb{H}^{r_1} \times \mathbb{H}^{r_2})_{s.s.}$   $\leftarrow$  s-stable part   
 i.e., corresponding to semi-stable  $\mathcal{O}_F$ -lattices  $\Lambda$ .

$\Rightarrow$  Considering arithmetic quotient  $\Gamma \backslash \mathbb{H}^{r_1} \times \mathbb{H}^{r_2}$  w/  $\Gamma = SL(\mathcal{O}_F \oplus \mathcal{O}_2)$

Q: Fundamental Domain.

### II. Cusps II.1. Definition

Working Site:  $SL(\mathcal{O}_F \oplus \mathcal{O}_2) \hookrightarrow SL_2(F) \ni M \ni \mathbb{H}^{r_1} \times \mathbb{H}^{r_2}$    
  $\begin{matrix} (u & v \\ c & d) \end{matrix} \quad (z, w) \mapsto \left( \frac{az+bx}{cz+dx}, \frac{azw+bx}{czw+dx} \right)$



$\Rightarrow$  Induced by an action  $SL_2(F) \curvearrowright \mathbb{P}^1(F)$  all compatible

$\Rightarrow$  Cusps of  $SL(\mathcal{O}_F \oplus \mathcal{O}_2) \backslash \mathbb{H}^{r_1} \times \mathbb{H}^{r_2}$

$=$  orbits of the action of  $SL(\mathcal{O}_F \oplus \mathcal{O}_2)$  on  $\mathbb{P}^1(F)$ .

Often, element in orbit  $=$  cusp.

$C_\Gamma := \{ \text{cusps of } \Gamma \}$

for simplicity.