- $\tilde{\Lambda} := \tilde{\Lambda}(P) :=$ space of O_F-lattices whose underlying O_F-module is P.

$\forall \sigma \in S_\infty$, $\tilde{\Lambda}_\sigma :=$ space of inner product on V_σ
with a chosen basis $\Rightarrow \tilde{\Lambda}_\sigma$ may be realized as an open subset of a $\mathbb{R}$ or $\mathbb{C}$ - vector space.

$\Rightarrow$ topology on $\tilde{\Lambda} = \prod_{\sigma \in S_\infty} \tilde{\Lambda}_\sigma$ (via measures.)

- $\forall \Lambda \in \tilde{\Lambda}$, $u, v \in \Lambda_\sigma$, $\langle u, w \rangle_{\Lambda, \sigma}$ or $\langle u, v \rangle_{g_\Lambda(\sigma)}$ the corresponding inner product for Λ.
- $A \in GL(P) \rightsquigarrow$ New lattice $A \cdot \Lambda$ in $\tilde{\Lambda}$ w/ $\langle u, w \rangle_{A\cdot\Lambda, \sigma} := \langle A^{-1}u, A^{-1}w \rangle_{\Lambda, \sigma}$.
& $\Lambda \simeq A \cdot \Lambda$ via $v \mapsto A \cdot v$ is an isometry.

Conversely, suppose $A : \Lambda_1 \cong \Lambda_2$ is an isometry of O_F-lattices
then A determines an element $A \in GL(P)$ s.t. $\Lambda_2 \cong A \cdot \Lambda_1$.

$\Rightarrow$ $GL(P) \backslash \tilde{\Lambda}(P)$: isometry classes of O_F-lattices $/P$.

- $\forall T \in \mathbb{R}_{>0}$, $\Lambda \mapsto \Lambda[T]$ new lattice obtained by multiplying $\underbrace{\Lambda_\sigma}_{\text{the inner product of}}$ by T^2.

$\Lambda := \Lambda(P) := \tilde{\Lambda} / \sim_{T\mathbb{R}_{>0}}$. natural topological structure.

- $P_1 \leq P$ submodule of P, $(P_1, g|_{P_1})$ a lattice $=: P_1 \cap \Lambda =: \Lambda_1$

Def. Λ_1 sublattice of Λ, $\Lambda_1 \leq \Lambda$, if P/P_1 is projective as well.

- $V := \Lambda \otimes_{\mathbb{Z}} \mathbb{R} = \prod_{\sigma \in S_\infty} \Lambda_\sigma$ Define an inner product on the real vector space $V/\mathbb{R}$
(O_F: an $\mathbb{Z}$-module is of rank $[F:\mathbb{Q}]$) by $\langle v, w \rangle_\infty := \sum_{\sigma:\mathbb{R}} \langle u_\sigma, w_\sigma \rangle_{g_\sigma}$ $(*)$,
$+ \sum_{\tau:\mathbb{C}} \mathrm{Re} \langle u_\tau, w_\tau \rangle_{g_\tau}$.

$\mathrm{Res}_{F/\mathbb{Q}} \Lambda$: the $\mathbb{Z}$-lattice obtained by equipping P, regarding as $\mathbb{Z}$-module, w/ this inner product $(*)$,

- $\mathrm{Vol}_{Leb}(\Lambda) :=$ (co)volume $(\mathrm{Res}_{F/\mathbb{Q}} \Lambda)$.

Example. $P = O_F$. & $\forall \sigma \in S_\infty$, $\{1\}$ an ON basis of $V_\sigma := F_\sigma$.

$\Rightarrow$ O_F as an O_F-lattice $\overline{O}_F := (O_F, \mathbb{1})$

$\left\{ \mathrm{Vol}_{Leb}(\overline{O}_F) = 2^{-r_2} \cdot |\Delta_F|^{\frac{1}{2}} \right.$.

Δ_F: discriminant of F

$O_F \hookrightarrow F \hookrightarrow \mathbb{R}^{r_1} \times \mathbb{C}^{r_2}$

$\mathbb{R}^{r_1} \times \mathbb{C}^{r_2}$: standard Euclidean