(c) <u>Arithmetic truncation</u>

$$\{\Lambda \hookrightarrow \mathbb{R}^2 : \text{rank 2 lattice of volume 1}\}/\sim \text{isometry}$$

$\updownarrow$ Reduction theory $\leftarrow$ classical

$$\mathcal{D}_{SL_2(\mathbb{Z})}$$

$\Rightarrow$ The points in D_T constructed in (a)

$\updownarrow$

rank 2 lattices of volume 1 whose 1^{st} Minkowski minimum λ_1 satisfying $\lambda_1^{-2} \le T$ or the same $\lambda_1 \ge \frac{1}{\sqrt{T}}$.

(i) $\Lambda \ni x_1 = (\lambda_1, 0)$

λ_1: 1st Minkowski minimum

i.e. $\lambda_1 = \min_{x \in \Lambda \setminus \{0\}} \|x\|$ + rotation

$\Downarrow$

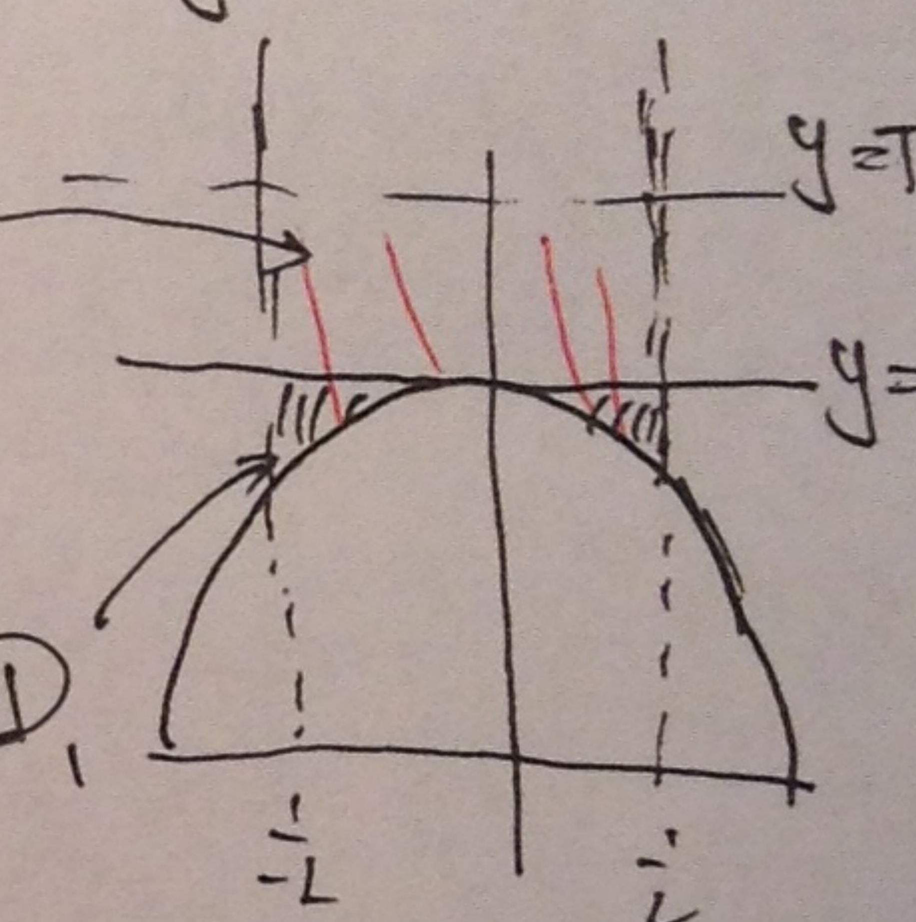
$\frac{1}{\lambda_1}\Lambda = \mathbb{Z} + \mathbb{Z}\tau$, $\tau = x_0 + i y_0 \in \mathcal{D}$ w/ $y_0 = \lambda_1^{-2}$.

$\Rightarrow$ set $\mathcal{M}_{\mathbb{Q},2}^{\le \frac{1}{2}\log T}$:= moduli space of rank 2 lattices Λ of volume 1 $/\mathbb{Q}$ whose sublattices Λ_1 of rank 1 have degree $\le \frac{1}{2}\log T$.

<u>Thm</u> (1) (Geometric truncation = Arithmetic truncation)

$$\mathcal{M}_{\mathbb{Q},2}^{\le \frac{1}{2}\log T}[1] \simeq D_T$$

In particular $\mathcal{M}_{\mathbb{Q},2}^{\le 0}[1] = \mathcal{M}_{\mathbb{Q},2}[1] \simeq D_1$



(2) (Special Uniformity of Zetas)

$$\hat{\zeta}_{\mathbb{Q},2}(s) = \hat{\zeta}_{\mathbb{Q}}^{SL_2}(s) = \frac{\hat{\zeta}(2s)}{s-1} - \frac{\hat{\zeta}(2s-1)}{s}.$$

Our aim of next 3 lectures:

What happens for general # field F?!

$\mathcal{O}_F$-lattices, fundamental domain, cusps, stabilizer groups of $SL_2(\mathcal{O}_F \oplus \mathfrak{a})$.

Eisenstein series,

Fourier Expansion of Eisenstein series, Rankin-Selberg Method

Stability & Distance to Cusps!!! ..