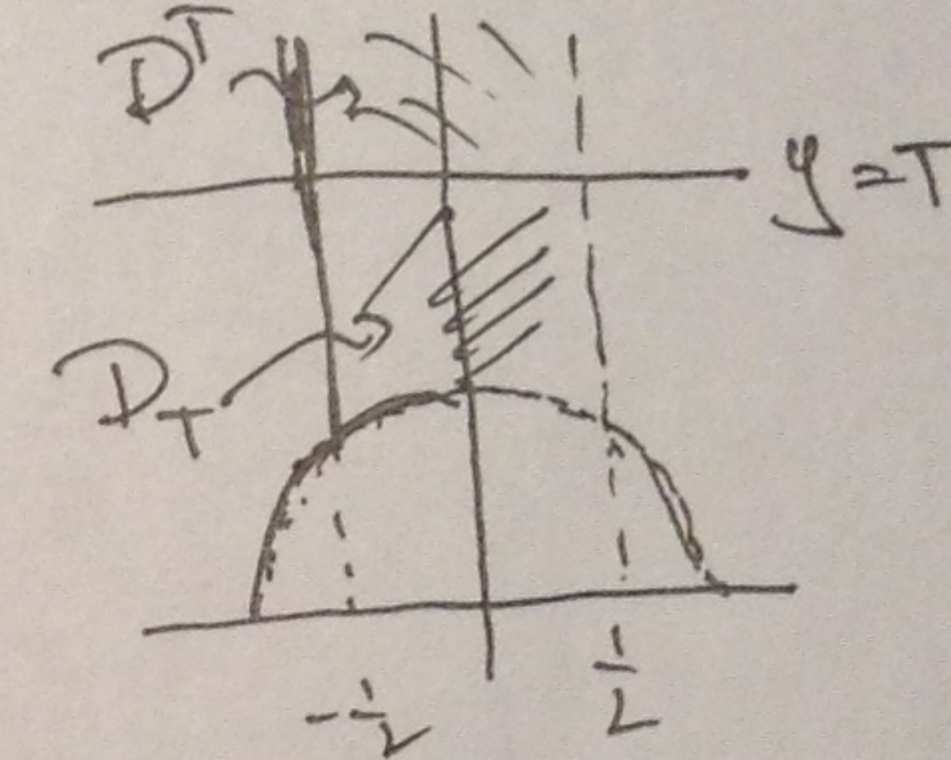
$\Rightarrow$ To make integrations meaningful, we need to cut-off the slow growth part.

Two ways to do so: $\begin{cases} \text{Geometric One: simple \& direct} \\ \text{Analytic one: technical \& traditional as Rankin-Selberg Method!} \end{cases}$

(a) Geometric Truncation



Let $T \geq 1$, set

$$D_T = \{ z \in \mathcal{D} \mid \operatorname{Im}(z) \leq T \}, \quad \mathcal{D}^T = \{ z \in \mathcal{D} \mid \operatorname{Im}(z) \geq T \}$$

$$\mathcal{D} = D_T \cup \mathcal{D}^T.$$

Introduce a well-defined integration

$$I_T^{geo}(s) := \int_{D_T} \hat{E}(z,s) \frac{dx \wedge dy}{y^2}$$

$\Rightarrow$ cpt domain + smooth function on z $\Rightarrow$ convergence of $I_T^{geo}(s)$.

(b) Analytic truncation

Define the truncated Eisenstein series $\hat{E}_T(z,s)$ by

$$\hat{E}_T(z,s) := \begin{cases} \hat{E}(z,s); & \text{if } y \leq T \\ \hat{E}(z,s) - a_0(y;s) & \text{if } y > T. \end{cases}$$

Introduce a well-defined integration

$$I_T^{ana}(s) := \int_{\mathcal{D}} \hat{E}_T(z;s) \frac{dx\,dy}{y^2}$$

$\Rightarrow$ exponentially decay at ∞ $\Leftarrow$ well-defined.

Thm (Rankin-Selberg)

$$I_T^{geo}(s) = \frac{\hat{\zeta}(2s)}{s-1} \cdot T^{s-1} - \frac{\hat{\zeta}(2s-1)}{s} T^{-s} = I_T^{ana}(s).$$

	Analytic One	= Geometric One	
Merit:	Fundamental Domain	Eisenstein	~~Kept~~ kept
	Eisenstein	Fundamental Domain	changed

?: Anything preserving Eisenstein series & Modular interpretation of Domain?!!