$\Rightarrow$ <u>Higher Sato-Tate Conj</u>: For non-CM elliptic curve $\mathcal{E}$ & $0 \leq \alpha < \beta \leq \pi$

$$\lim_{x \to \infty} \frac{\#\{p : \text{prime},\ p \leq x,\ \sin(\frac{\pi}{2} - \alpha) \geq \Theta^{\mathcal{E}}_{n,p} \geq \sin(\frac{\pi}{2} - \beta)\}}{\#\{p : \text{prime},\ p \leq x\}}$$

$$= \frac{2}{\pi} \int_\alpha^\beta \sin^2\theta \cdot d\theta$$

Here $\Theta^{\mathcal{E}}_{n,p} := \underbrace{\frac{p^{\frac{n-1}{2}}}{n-1}}_{\text{huge factor}} \cdot \underbrace{\left(\frac{\pi}{2} - \theta^{\mathcal{E}}_{n,p}\right)}_{\text{limit}=0} + \underbrace{\frac{1}{2}\left(\sqrt{p} + \frac{1}{\sqrt{p}}\right)}_{\text{normalization additive factor.}}$ for $n p \to \infty$.

<u>Lemma</u>: Sato-Tate $\Rightarrow$ Higher Sato-Tate

$\because$ (I) of W-Z.

But Don does not like this formulation. Not natural, he said.

2.2. Number Field

For simplicity, $F = \mathbb{Q}$ for $\hat{\zeta}_{F,n}(s)$ Non-Abelian zeta function.
(even not yet defined, but ...)

<u>Assm</u>$^{(\text{WConj})}$ (Weak RH) Outside a finite box, $\mathrm{Re}(s) = \frac{1}{2}$ if $\hat{\zeta}_{F,n}(s) = 0$.

Assuming RH. set

$$N^n(T) = \#'\{s = \tfrac{1}{2} + i\gamma,\ \hat{\zeta}_{\mathbb{Q},n}(s) = 0,\ 0 < \gamma < T\}$$

<u>Conj 1</u>$_n$ $\cancel{N^n_{\mathbb{Q}}(2\pi T)}$ $N^n_{\mathbb{Q}}(2\pi T) = nT \cdot \log T - nT + O\left(\frac{\log T}{(\log^2 T)}\right)$

Let $\cdots < \gamma^n_0 < \gamma^n_1 < \gamma^n_2 < \cdots$ imaginary part of zeros of $\hat{\zeta}_{\mathbb{Q},n}(s)$, $\nearrow$ order
$\gamma^n_0 = 0$

<u>Cor</u>: Conj 1 $\Rightarrow$ $\gamma^n_k = \frac{2\pi}{n} \cdot \frac{k}{\log k}\left(1 + O\left(\frac{1}{\log k}\right)\right)$.