2. Distributions.

2.1. Sato-Tate Conjecture.

$E/\mathbb{F}_q$: elliptic curve $\quad N(E) := \# E(\mathbb{F}_p)$.

RH $\Rightarrow$ $\quad \cos\theta_p := \dfrac{p+1-N(E/\mathbb{F}_p)}{2\sqrt{p}}$ $\quad 0<\theta<\pi$ makes sense.

<u>Conj</u> (Sato-Tate Conj) if $\{E(\mathbb{F}_p)\}$ are not (resulting from a global one) of CM type, then for $0 \le \alpha < \beta \le \pi$

$$\lim_{x\to\infty} \frac{\#\{p \text{ prime}: p \le x,\ \alpha \le \theta_p \le \beta\}}{\#\{p \text{ prime}: p \le x\}} = \frac{2}{\pi}\int_\alpha^\beta \sin^2\theta \, d\theta.$$

Motivated by this,

Set
$$\cos\theta_{n,p}^E := \frac{(p^n-1)\,\beta_{E,n}(0) + (p^n+1)\,\alpha_{E,n}(0)}{2\sqrt{p^n}\,\alpha_{E,n}(0)} \overset{\text{CM}}{=} \frac{-(p^n-1)\cdot\frac{\beta_n}{\beta_{n-1}} + (p^n+1)}{2\sqrt{p^n}}$$

<u>Thm</u>
$$\lim_{x\to\infty} \frac{\#\{p: \text{prime } p\le x,\ \alpha \le \theta_{n,p}^E \le \beta\}}{\#\{p: \text{prime } p \le x\}} = \begin{cases} \int_\alpha^\beta \delta_{\frac{\pi}{2}}\, dt, & n\ge 3 \\ \int_\alpha^\beta \delta_{\frac{\pi}{3}}\, dt, & n=2 \end{cases}$$

$\because$) $\displaystyle\lim_{p^n\to\infty} \cos\theta_{n,p}^E = 0$

Rather simple & totally disappointed by this in early stage.

<u>But</u>: Not right to stop here!!!

<u>Thm</u> (W-Z).
$$\frac{\beta_n}{\beta_{n-1}} = 1 + \frac{(n-1)\cdot(N(E/\mathbb{F}_q) - c(q))}{q^n} + O\left(\frac{n^2}{q^{2n-2}}\right) \quad (*)$$

as $q^n \to \infty$

w/ $c(q) = 2 + \dfrac{3(a-2)}{q} + \cdots$ independent of n, $a = q+1-N(E/\mathbb{F}_q)$.

Dominant term $\rightsquigarrow \delta_{\frac{\pi}{2}}$

Should go to Subdominate term!!!