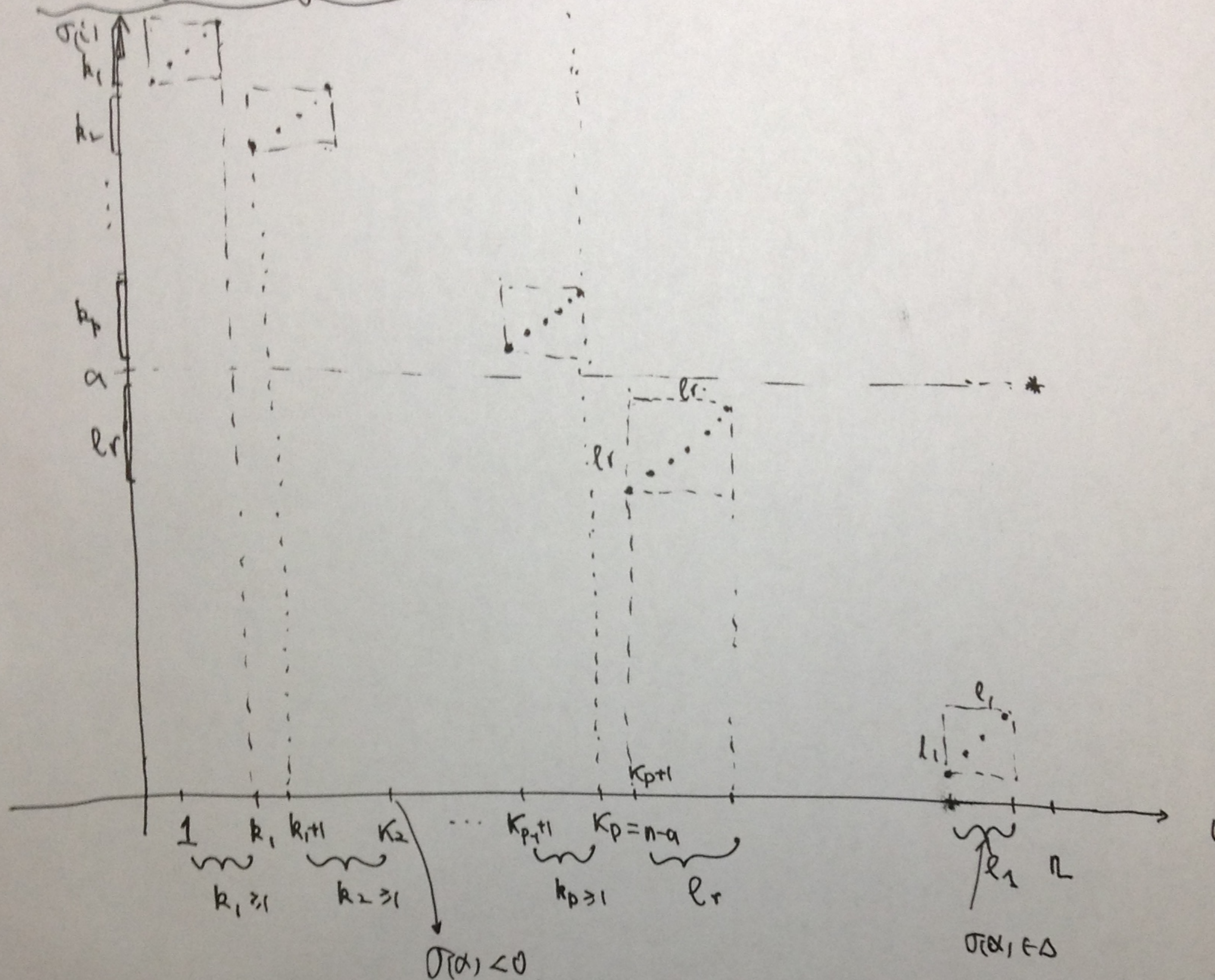
Characterization of Special Permutations



$\sigma = \sigma(k_1, k_2, \ldots k_p;\ a;\ l_r, \ldots, l_2, l_1)$ $\qquad a = \sigma(n).$

ordered partition: $\quad n-a = k_1+k_2+\cdots+k_p$

$\qquad a-1 = l_r+\cdots+l_2+l_1$

$\frac{\#\{a_i \geq 1\}}{q-1}$

$v_k = \hat{\zeta}_{\Sigma}^{(1)}\cdot\hat{\zeta}_{\Sigma}^{(2)}\cdots\hat{\zeta}_{\Sigma}^{(k)}$

Contribution to SL_n-zeta. (Wang-Zagier)

s: totally $\hat{\zeta}^{SL_n}_{\Sigma}(s) =$

$$\sum_{a=1}^{n}\ \sum_{k_1+\cdots+k_p=n-a} \frac{v_{k_1}\cdots v_{k_p}}{(1-q^{k_1+k_2})\cdots(1-q^{k_{p-1}+k_p})}\cdot\frac{1}{1-q^{ns-n+a+k_p}}$$

$$\times \sum_{l_r+\cdots+l_1=a-1} \frac{v_{l_r}\cdots v_{l_1}}{(1-q^{l_r+l_{r-1}})\cdots(1-q^{l_2+l_1})}\cdot\frac{1}{1-q^{-ns+n-a+1+l_r}}$$

$$= \frac{\hat{\zeta}^{(s)}_{\Sigma,n}}{q^{\frac{n-1}{2}n(g-1)}}.$$