Def. $\hat{\zeta}^{SL_n}_{X}(s) := \hat{\zeta}^{(SL_n, P_{n-1,1})}_{X}(-ns) \Rightarrow \boxed{\hat{\zeta}^{SL_n}_{X}(s) = \hat{\zeta}^{SL_n}_{X}(1-s)}$ ← F.E.

Thm. (Special Uniformity of Zetas)

$$\hat{\zeta}_{X,n}(s) = \zeta^{\overset{\dim R_u(B)\cdot(g-1)}{\underset{\frac{n(n-1)}{2}}{\shortparallel}}} \cdot \hat{\zeta}^{SL_n}_{X}(s).$$

Two ways of proof. $\begin{cases} \text{Periods: "Weng's ... Mellin transform."} \\ \text{Wall-crossing: Mozgovoy-Reineke-W.-Z.} \end{cases}$

- Explicit Expression.

Easily. $\omega^{G,P}_{X}(-ns) = \lim_{\lambda \to (ns-n)\varpi_p + \rho} \left(\prod_{\alpha \in \Delta_p} (1 - q^{-\langle \lambda - \rho, \alpha^\vee \rangle}) \cdot \omega^{SL_n}_{X}(\lambda) \right)$

Each $w \in W$

$$\lim_{\lambda \to (ns-n)\varpi_p + \rho} \frac{\prod_{\alpha \in \Delta \setminus \{\alpha_{n-1}\}} 1 - q^{\overset{-\langle\lambda,\alpha\rangle+1}{\overset{\shortparallel}{-\langle \lambda-\rho, \alpha^\vee\rangle}} = 0}}{\prod_{\alpha \in \Delta} 1 - q^{\underset{\underset{\langle \lambda, w^{-1}\alpha^\vee\rangle + 1}{\shortparallel}}{-\langle w\lambda - \rho, \alpha^\vee \rangle}}} \cdot \prod_{\substack{\alpha>0 \\ w\alpha<0}} \frac{\hat{\zeta}_X(\langle \lambda, \alpha^\vee \rangle)}{\hat{\zeta}_X(\langle \lambda, \alpha^\vee \rangle + 1)} \neq 0.$$

$\Rightarrow$ Cancelation w/ other factors

otherwise $= 0$.

$\begin{cases} ① \text{ Denominator} \quad w^{-1}\alpha \in \Delta_p. \\ ② \text{ poles of zeta } \hat{\zeta}_X(\langle \lambda, \alpha \rangle) \underset{\text{i.e. } 1}{\shortparallel} \quad \alpha > 0,\ w\alpha < 0 \ \&\ \alpha \in \Delta_p. \end{cases}$

$W_0 = \{ w \in W \mid \{\alpha \in \Delta_p : w\alpha < 0 \text{ or } w\alpha \in \Delta\} = \Delta_p \} \longleftrightarrow S_n^0 \ni \sigma$

special permutations