Hence

$$\mathbb{L}^{\langle \rho_Q^Q, \nu_\alpha \rangle + \dim(N_Q)(g-1)} \cdot \mu(M_{X, M_Q}(\nu_Q')) = \sum_{P: P\subset Q} (-1)^{\dim \mathfrak{a}_P^Q} \sum_{\substack{\nu_P' \in X_*(A_P') \\ [\nu_P']_Q = \nu_Q}} \hat{\tau}_P^Q([\nu_P']^Q) \cdot \mu(M^{total}_{X, M_P}(\nu_P'))$$

$$\mu(M^{total}_{X, G, Q}(\nu_Q')) = \sum_{P: P\subset Q} \sum_{\substack{\nu_P' \in X_*(A_P') \\ [\nu_P']_Q = \nu_Q}} \hat{\tau}_P^Q([\nu_P']^Q) \, \mathbb{L}^{\langle 2\rho_P^Q, \nu_P' \rangle + \dim(N_P)\cdot(g-1)} \cdot \mu(M_{X, M_P}(\nu_P')).$$

Set

$$\tilde{\mu}(M_{X, M_P}(\nu_P')) = \mathbb{L}^{\dim(N_P)(g-1)} \cdot \mu(M_{X, M_P}(\nu_P'))$$

$$\tilde{\mu}(M^{total}_{X, M_P}(\nu_P')) = \mathbb{L}^{\dim(N_P)\cdot(g-1)} \cdot \mu(M^{total}_{X, M_P}(\nu_P'))$$

Main Thm

$$\tilde{\mu}(M^{total}_{X, M_Q}(\nu_Q')) = \sum_{P: P\subset Q} \sum_{\bar{\nu}_P' \in (\Lambda_P^Q)^\perp} \tilde{\mu}(M_{X, M_P}(\nu_P')) \sum_{\substack{\bar{\omega} \in \Pi_P^Q \\ [\bar{\omega}]_Q = \nu_Q}} \prod_{\alpha \in \Delta_P^Q} \frac{\mathbb{L}^{2\langle \rho_P^Q, \bar{\omega}_\alpha^\vee \rangle \cdot \{\alpha^\vee_{\nu_Q}(\bar{\omega})\}}}{\mathbb{L}^{2\langle \rho_P^Q, \bar{\omega}_\alpha^\vee \rangle} - 1}$$

$$\tilde{\mu}(M_{X, M_Q}(\nu_Q')) = \sum_{P: P\subset Q} (-1)^{\dim \mathfrak{a}_P^Q} \tilde{\mu}^{total}_{X, M_P}(0) \sum_{\substack{\lambda \in \Lambda_P^Q \\ [\lambda]_Q = \nu_Q}} \prod_{\alpha \in \Delta_P^Q} \frac{\mathbb{L}^{2\langle \rho_P^Q, \alpha^\vee \rangle \cdot \{\bar{\alpha}^Q_\alpha(\lambda)\}}}{\mathbb{L}^{2\langle \rho_P^Q, \alpha^\vee \rangle} - 1}$$

Here

$$\Lambda_P^Q \doteq X_*(A_P') / \sum_{\alpha \in \Delta_P^Q} \mathbb{Z}\alpha^\vee$$

$$\Pi_P^Q \doteq X_*(A_P') / \sum_{\alpha \in \Delta_P^Q} \mathbb{Z}\bar{\omega}_\alpha^\vee.$$

$\forall \lambda \in \mathbb{R}/\mathbb{Z}$ & $0 < \langle \lambda \rangle \le 1$, $0 \le \{\lambda\} = 1 - \langle \lambda \rangle = x - [x] < 1$.

$\because$)

$$\sum_{n\in\mathbb{Z}, n+x>0} t^n = \frac{t^{\langle x+\mathbb{Z}\rangle - x}}{1-t} \quad \#.$$

③ Levi factors & their simple components.

G: connected reductive group $/K$, Z: maximal central K-torus, $G' = D(G)$: semi-simple derived group. G_i: K-simple factor of G'.

$\Rightarrow$ Central isogeny $Z \times \prod_i G_i \to G$

& $\mu(M^{total}_{X, Z\times\prod G_i}(\nu_G)) = \hat{\zeta}_X^{\wedge}(1)^{?} \cdot \prod_{i\in I} \mu(M^{total}_{X, G_i})$. ← semi-simple.

- (central) Isogeny for torus ← Ono's result + Lang. ↷ with {residue field}.
- semisimple group ↘ Galois extension. Standard.
-

④ Semi Simple Groups

$n_i = \#\{\alpha \in \Phi^+ : \langle \rho, \alpha \rangle = i\} - \#\{\alpha \in \Phi^+ : \langle \rho, \alpha \rangle = i+1\}$

$$\mu(M^{total}_{X, G}(0)) = \mathbb{L}^{\cdot} \prod_i \hat{\zeta}_X(i)^{-n_i}$$

essentially Langlands. [Kim-Weng].