

PCQ $\Rightarrow \begin{cases} \mathbb{P}_P^Q := \mathbb{P}_{P \cap M_\alpha} = \{ \alpha \in \mathbb{P}_P \text{ occurring in } M_\alpha \} \\ \mathbb{P}_P^{QT} := \mathbb{P}_{P \cap M_\alpha}^T = \{ \alpha \in \mathbb{P}_P^T \} \\ \Delta_P^\alpha := \Delta_{P \cap M_\alpha} = \{ \alpha \in \Delta_P \} \end{cases} \Rightarrow \begin{cases} (i) \Delta_P^\alpha \subset \mathbb{P}_P^{\alpha^*} \text{ basis} \\ (ii) \text{the projection of } (\alpha^v)_{\alpha \in \Delta_P^\alpha} \text{ into } \mathbb{P}_P^\alpha \text{ basis of } \mathbb{P}_P^\alpha \\ \downarrow \text{dual basis} \\ (iii) \{ \omega_\alpha^\alpha \}_{\alpha \in \Delta_P^\alpha} \subset \mathbb{P}_P^{\alpha^*} \text{ basis as well.} \end{cases}$

Acute Weyl chamber

$$\mathbb{O}_P^{\alpha^+} := \{ H \in \mathbb{O}_P^\alpha : \langle \alpha, H \rangle > 0 \forall \alpha \in \Delta_P^\alpha \} \Rightarrow \mathbb{Z}_P^\alpha$$

Obtuse Weyl chamber

$$\mathbb{O}_P^\alpha := \{ H \in \mathbb{O}_P^\alpha : \langle \tilde{\omega}_\alpha^\alpha, H \rangle > 0 \forall \alpha \in \Delta_P^\alpha \} \Rightarrow \hat{\mathbb{Z}}_P^\alpha$$

Lemma: $\mathbb{O}_P^{\alpha^+} \subset \mathbb{O}_P^{\alpha^-}$

Lemma (Langlands)

$\forall P \in R \quad \forall H \in \mathbb{O}_P^R$

then
$$\sum_{Q: P \subset Q \subset R} (+) \dim \mathbb{O}_Q^R \hat{z}_P^Q([H]^\alpha) \hat{z}_Q^R([H]^\alpha) = \int_P^R$$

$$\sum_{Q: P \subset Q \subset R} (+) \dim \mathbb{O}_Q^\alpha \hat{z}_P^Q([H]^\alpha) \hat{z}_Q^R([H]^\alpha) = \int_P^R$$

Constant term:
$$S_P(g) = \int_{N_P(\mathbb{R}) \backslash N_P(\mathbb{A})} S(ny) dn \quad \forall T \in \mathbb{O}_P$$

Arthur's truncation:
$$(\Lambda^T S)(g) = \sum_P (+) \dim(\mathbb{O}_P) \sum_{\delta \in P(\mathbb{R}) \backslash P(\mathbb{A})} S_P(\delta g) \cdot \hat{z}_P([H(\delta g)] - T)$$

Prop. \forall nice system of functions $\{ a(\alpha, v_\alpha') \}_{(\alpha, v_\alpha') \in \beta} \quad v_\alpha' \in \Sigma_\alpha(A_\alpha) \cdot \{ b(\alpha, v_\alpha') \}_{(\alpha, v_\alpha') \in \beta}$

$$a(\alpha, v_\alpha') = \sum_{\substack{P \in \mathcal{P} \\ P \subset \alpha}} \sum_{\substack{v_p' \in \Sigma_\alpha(A_p) \\ \langle v_p', \alpha \rangle = v_\alpha'}} z_p^\alpha([v_p']^\alpha) \cdot b(P, v_p')$$

$$\Leftrightarrow b(\alpha, v_\alpha') = \sum_{P: P \subset \alpha} (+) \dim(\mathbb{O}_P^\alpha) \sum_{\substack{v_p' \in \Sigma_\alpha(A_p) \\ \langle v_p', \alpha \rangle = v_\alpha'}} \hat{z}_P^\alpha([v_p']^\alpha) a(P, v_p')$$

Langlands - Rapoport