(I)

$\mathcal{E}_P/X$: principal P-bundle on X

$\mu(\mathcal{E}_P)$: slope of $\mathcal{E}_P$ $\doteq$ $\forall \xi \in X^*(A_P')$ (character) $\langle \xi, \mu(\mathcal{E}_P)\rangle = c_1(\mathcal{E}_{P,\xi}) \in \mathbb{Z}$

$\in X_*(A_P')$ 1ps of A_P' (maximal split torus of M_P^{ab}, quotient)

$P \twoheadrightarrow A_P' \xrightarrow{\xi} \mathbb{G}_m = GL_1$

Semi-stable: $\forall Q \subset P$ s.t. $|\Delta_Q^P| = 1$ $\rightsquigarrow \Delta_Q^P = \{\alpha_Q^P\}$ (simple element)

$\forall \mathcal{E}_Q/Q$-principal bdl s.t. $\mathcal{E}_P \cong \mathcal{E}_Q \times^Q P$

$\langle \alpha_Q^P, \mu(\mathcal{E}_Q)\rangle \le 0$. $\boxed{Q \to P}$

Lem. $\forall$ G-bdl $\mathcal{E}$ of slope $\nu_G' \in X_*(A_G')$, !

(i) $\exists! (P, \nu_P')$ w/ $\begin{cases} [\nu_P']_G = \nu_G' \\ [\nu_P']^G \in \mathfrak{a}_P^{G,+} \end{cases}$ (ii) s.stable P-bdl $\mathcal{E}_P$ of slope ν_P' $P \to G$

s.t. $\iota : \mathcal{E}_P \times^P G \simeq \mathcal{E}$.

Moreover the pair (P, ν_P') & the isomorphism class of the pair $(\mathcal{E}_P, \iota)$ are uniquely determined by $\mathcal{E}$.

准備: $\mathfrak{a}_P = \mathbb{R} \otimes X_*(A_P) = \mathbb{R} \otimes X_*(A_P')$ $\forall P \supset Q$ $A_Q \hookrightarrow A_P \hookrightarrow A_P' \twoheadrightarrow A_Q'$.

$\mathfrak{a}_Q \hookrightarrow \mathfrak{a}_P$ $\mathfrak{a}_P \twoheadrightarrow \mathfrak{a}_Q$

$\mathfrak{a}_P = \mathfrak{a}_P^Q \oplus \mathfrak{a}_Q^R \oplus \mathfrak{a}_R$ ($\mathfrak{a}_P^Q$, $\mathfrak{a}_Q^R$, $\mathfrak{a}_R$; $[\ ]^Q$, $[\ ]^R$, $[\ ]_R$)

$\Downarrow$

$\mathfrak{a}_P^* = \mathfrak{a}_P^{Q*} \oplus \mathfrak{a}_Q^{R*} \oplus \mathfrak{a}_R^*$ $\Big\}$ $\mathfrak{a}_P^* = \mathfrak{a}_P^{Q*} \oplus \mathfrak{a}_Q^*$ $\Leftarrow$ $\mathfrak{a}_P = \mathfrak{a}_P^Q \oplus \mathfrak{a}_Q$.

- $\Phi_P \in \mathfrak{a}_P^{G*} \subset \mathfrak{a}_P^*$: the set of the non-trivial characters of A_P occurring in $\mathfrak{g}$ (Lie G)
- $\Phi_P^+ \subset \Phi_P$: the set of the non-trivial characters of A_P occurring in $\mathfrak{n}_P$ = Lie N_P (nilpotent radical)
- $\Phi_0 := \Phi_{P_0}$: root system $\supset \Phi_0^+$: order positive roots, $\Delta_0 = \Delta_{P_0} \subset \Phi_0^+$ simple roots.
- $\alpha \in \Phi_0 \rightsquigarrow \alpha^\vee = \frac{2}{\langle \alpha, \alpha\rangle}\alpha$: coroot $\leftarrow$ $\{\alpha^\vee\}_{\alpha \in \Delta_0}$: basis of $\mathfrak{a}_0^G \subset \mathfrak{a}_0$ ↖ basis of $(\mathfrak{a}_{P_0}^G)^*$.

($\mathfrak{a}_P^* \simeq \mathfrak{a}_P$ under natural pairing)

- Φ_P : root a root system $\Phi_P^+ = \Phi_P \cap \Phi^+$ & $\supset \Delta_P := \{$non-trivial restriction to A_P (or $\mathfrak{a}_P$) of simple roots $\in \Delta_0\}$

$\Rightarrow \Delta_P$: basis of $\mathfrak{a}_P^{G*}$

& $\forall \alpha \in \Delta_P$ $\exists$ coroot $\alpha^\vee \in \mathfrak{a}_P^G$ w/ property that: $(\alpha^\vee)_{\alpha \in \Delta_P}$: basis of $\mathfrak{a}_P^G$

defined as follows: $\alpha = \beta|_{A_P}$ ($\exists! \beta \in \Delta_0$) $\rightsquigarrow$ $\alpha^\vee := \beta^\vee|_{\mathfrak{a}_P^G}$.

$\begin{cases} \rho_P^G = \frac{1}{2}\sum_{\alpha \in \Phi_P^{G+}} \alpha \\ \rho = \frac{1}{2}\sum_{\alpha \in \Phi^+} \alpha \end{cases}$

VI–VIII. SL_2.

(IX) Parabolic Reduction, Stability & the Masses. (予定)

I. Motivic Euler Product & Applications

II. Non-Abelian Special Uniformity of Zetas: $\hat{\zeta}_{X,n} = \hat{\zeta}_X^{SL_n}$ (?)

III. RH/Elliptic curves. IV. Shape Duality & Cohen-Hensel.

V. Periods, Coded Field