IV. Total & Stable Masses

(5)

$\forall g \in G(\mathbb{A})$, $\mathrm{Aut}(g) := gKg^{-1} \cap G(F)$.

$$\beta^{total}_{X,G}(\nu_G') := \int^{\#}_{G(F)\backslash G(\mathbb{A})^{(\nu_G')}/K} \frac{1}{\mu(\mathrm{Aut}(g))} |\omega_{\mathbb{A}}|$$

twist of $G(\mathbb{A})^1$

$$\beta_{X,G}(\nu_G') := \int^{\#}_{(G(F)\backslash G(\mathbb{A})^{(\nu_G')}/K)_{c.stable}} \frac{1}{\mu(\mathrm{Aut}(g))} |\omega_{\mathbb{A}}|$$

$\mathcal{M}^{(\nu_G)}_{X,G}$

Fact. $\exists$ natural morphism

$$G(F)\backslash G(\mathbb{A})^{(\nu_G')}/K \to \mathcal{M}^{total}_{X,G}(\nu_G')$$

stack of G-torsor of degree $\nu_G' \in X_*(A_G)$.

Aim. structures of β's!!!

$$\tau(G) = \int_{G(F)\backslash G(\mathbb{A})^1} |\omega_{\mathbb{A}}| = \sum^{\omega}_{[g]} \int_K |\omega_{\mathbb{A}}| \cdot \frac{1}{\mu(g^{-1}Kg \cap G(F))}$$

$$= \mu(K) \cdot \sum^{\omega}_{[g]} \frac{1}{\mu(g^{-1}Kg \cap G(F))} = \mu(K) \cdot \beta^{total}_{X,G}(\nu_G').$$

$$\coprod_{[g]} \frac{G(F)gK}{G(F)} \simeq K/(gKg^{-1} \cap G(F))$$

Thm (i).

$$\beta^{total}_{X,G}(\nu_G') = \frac{\tau(G)}{\mu(K)} = \tau(G) \cdot \mathbb{L}^{(1-g)\dim R_u(B)} \cdot \hat{Z}_X(M)^{-1}$$

where $\hat{Z}_X(u) := u^{(g-1)} Z_X(u)$ s.t. $\hat{Z}_X(u) = \hat{Z}_X\left(\frac{1}{qu}\right)$.

(ii)

$$\tilde{\beta}_{X,G}(\nu_G') = \sum_{P:\ \text{standard parabolic}} \mathrm{sgn}(P) \sum_{\nu_P' \mapsto \nu_G'} c_P(\nu_P')\, \tilde{\beta}^{total}_{X,M}(0)$$

depending only on root system

can be determined explicitly.

(iii)

$$\hat{\beta}^{total}_{X,G}(\nu_G') = \sum_{P:\ \text{stand. para}} \sum_{\nu_P' \mapsto \nu_G'} \rho(\nu_P')\, \beta_{X,M}(\nu_P') \leftarrow \text{finite sum}$$