Arithmetic Riemann-Roch Theorem:

An Approach with relative Bott-Chern Secondary Characteristic Objects A Sketch

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Introduction

In this paper, we will prove the arithmetic Riemann-Roch theorem for l.c.i. morphisms of arithmetic varieties. Roughly speaking, this arithmetic version of Grothendieck Riemann-Roch theorem is a direct consequence of the existence of relative Bott-Chern secondary characteristic objects for both smooth morphisms and closed immersions, which are developed in [W 91a] and [BGS 91] respectively.

It is well-known that Riemann-Roch theorem has a long history. The most important event of this theory began with Hirzebruch's remarkable formula. Soon after the discovery of Hirzebruch Riemann-Roch formula, we came to its two deep generalizations. The first is Grothendieck Riemann-Roch theorem in algebraic geometry, which states that the Riemann-Roch theorem in algebraic geometry is in fact the following commutative diagram:

$$\begin{array}{ccc} K_0(X) & \xrightarrow{\mathrm{ch}(\ )\mathrm{td}(f)} & \mathrm{CH}(X)_{\mathbf{Q}} \\ \downarrow f_K & & \downarrow f_{\mathrm{CH}} \\ K_0(Y) & \xrightarrow{\mathrm{ch}} & \mathrm{CH}(Y)_{\mathbf{Q}} \end{array}$$

for any l.c.i. morphism of regular varieties $f : X \to Y$. The second is the famous Atiyah-Singer index theorem, which deals with more general spin-manifolds and elliptic operators. In the recent years, following Seeley, Patodi and others, mathematians use heat kernel technique to offer a kind of local index theorem, instead of using corbodism theory. For more details, please see [B 86], [BS 58], [FL 85], [H 56], [SGA 6], and the collection of Atiyah's works [A].

Roughly speaking, the arithmetic Riemann-Roch theorem is a natural generalization of classical Riemann-Roch theorem in the sense of Grothendieck. In fact, if we use the arithmetic notations in the above diagram, we still have a commutative diagram for a l.c.i. morphism of regular arithmetic varieties. Here we add a technical assumption that the morphism at infinity should be smooth. The aim of this paper is to show a method to do so.

By the works of Gillet and Soulé, we know that at first we need to give a good definition for the push-out morphism of arithmetic K-groups. The basic strategy for offering this push-out morphism of arithmetic K-groups is to factor the l.c.i. morphism as a regular closed immersion followed by a projection as usual. But instead of using a push-out morphism of arithmetic K-groups for closed immersion, we directly offer one for l.c.i. morphism. In certain sense, this approach saves us lots of space, as to give a proper push-out morphism of arithmetic K-groups for closed immersion needs quite a big theory associated with relative version for arithmetic intersection theory and arithmetic K-theory. The key point in our definition is the theory about relative version of classical Bott-Chern secondary characteristic forms for both smooth morphisms and closed immersions [BGS 91] and [W 91a]. After we give this definition by using the properties of relative Bott-Chern secondary characteristic objects, the arithmetic Riemann-Roch theorem is a direct consequence.

One more thing we need to explain here is that indeed, in the category of complex geometry, arithmetic Riemann-Roch theorem is a refined version of classical Riemann-Roch: the classical theorem deals with algebraic cycles, but the arithmetic theorem deals with forms. Furthermore, our first axioms for the relative Bott-Chern secondary characteristic forms with respect to smooth forms is equivalent to local index theorem.

This paper is organized as follows: In part one, we give the existence theorem of relative Bott-Chern secondary characteristic objects for both smooth morphism and closed immersion. In part two, we will give the arithmetic Riemann-Roch theorem for l.c.i. morphism $f : X \to Y$, but with a technical assumption that f at infinity is smooth and

X, Y are regular.

As a sketch version of [W 91a] and [W 91b], here we only state the main results and the ideas behind our proof. The details may be found in [W 91a] and [W 91b].

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Part I. Relative Bott-Chern Secondary Characteristic Objects

In this part, we will give the axioms for various kinds of Bott-Chern secondary characteristic objects and state their existences. It contains three sections. In section 1, we deal with the classical Bott-Chern secondary characteristic forms. In section 2, we consider the relative Bott-Chern secondary characteristic forms for smooth morphism. Finally, in section 3, we give the theory about relative Bott-Chern secondary characteristic currents for closed immersion, following [BGS 91].

§I.1. Classical Bott-Chern Secondary Characteristic Forms

In this section, we recall the basic facts associated with the classical Bott-Chern secondary characteristic forms. References are [BC 68] and [BGS 88].

Let X be a complex manifold. A hermitian vector sheaf on X is a pair $(\mathcal{E}, \rho)$ consisting of a holomorphic vector sheaf $\mathcal{E}$ on X and a smooth hermitian metric ρ on $\mathcal{E}$. It is a standard fact that there exists a unique smooth connection $\nabla_{\mathcal{E}}$ on $\mathcal{E}$ which is unitary and the antiholomorphic component $\nabla_{\mathcal{E}}^{0,1}$ of which is equal to the Cauchy-Riemann operator $\overline{\partial}_{\mathcal{E}}$ of $\mathcal{E}$. As usual, we call this connection the hermitian holomorphic connection of $(\mathcal{E},\rho)$. Its curvature $K_{\mathcal{E}}$ is defined by $\nabla_{\mathcal{E}}^2$. Obviously, $K_{\mathcal{E}} \in \mathcal{A}^{1,1}(X, \mathrm{End}\mathcal{E})$. We let $R_{\mathcal{E}} = \frac{1}{2\pi i} K_{\mathcal{E}}$ be its Ricci tensor.

Let $B \subset \mathbf{R}$ be a subring, $\phi \in B[[T_1, ..., T_n]]$ be any symmetric power series. For every $k \geq 0$, let $\phi^{(k)}$ be the homogenous component of ϕ of degree k. Then there exists a unique polynomial map $\Phi^{(k)} : M_n(\mathbf{C}) \to \mathbf{C}$ such that

1. $\Phi^{(k)}$ is invariant under conjugation by $\mathrm{Gl}_n(\mathbf{C})$.
2. $\Phi^{(k)}(\mathrm{diag}(a_1, ..., a_n)) = \phi^{(k)}(a_1, ..., a_n)$.

More generally, for any B-algebra A, we can also define $\Phi = \bigoplus_{k\geq 0} \Phi^{(k)} : M_n(A) \to A$. Furthermore, if $I \subset A$ is a nilpotent subalgebra, then we may define

$$\Phi = \bigoplus_{k\geq 0} \Phi^{(k)} : M_n(I) \to A.$$

With this, if $(\mathcal{E},\rho)$ is a rank n hermitian vector sheaf on X, and ϕ as above, we can define

$$\phi(\mathcal{E},\rho) := \Phi(-R_{\mathcal{E}}) \in \mathcal{A}(X) := \bigoplus_{p\geq 0} \mathcal{A}^{p,p}(X)$$

as follows.

First locally identify End $\mathcal{E}$ with $M_n(\mathbf{C})$ and then apply the discussion above to

$$I = \oplus_{p\geq 1} \mathcal{A}^{p,p}(X).$$

Note that since Φ is invariant under conjugation, we have the following

Facts: 1. $\phi(\mathcal{E},\rho)$ is closed.

2. For any morphism f, $f^*(\phi(\mathcal{E},\rho)) = \phi(f^*\mathcal{E}, f^*\rho)$.

3. The de Rham cohomology class of $\phi(\mathcal{E},\rho)$ does not depend on the choice of ρ, but the form $\phi(\mathcal{E},\rho)$ does depend on ρ.

Fact 3 is the starting point for us to introduce the whole story below. Roughly speaking, the refined version of $\phi(\mathcal{E},\rho)$ for finite dimensional vector bundle and infinite dimensional vector bundle, i.e. Bott-Chern Secondary Characteristic Forms and Relative Bott-Chern Secondary Characteristic Forms, are central parts of our theory.

Next we state the following axioms for the **classical Bott-Chern secondary characteristic form,** $\phi_{\rm BC}(\mathcal{E}.,\rho_1,\rho_2,\rho_3)$ associated with a short exact sequence of vector sheaves $\mathcal{E}. : 0 \to \mathcal{E}_1 \to \mathcal{E}_2 \to \mathcal{E}_3 \to 0$ and with metrics ρ_j's on $\mathcal{E}_j$'s for $j = 1,2,3$.

Axiom 1. (Downstairs Rule) Let $\mathcal{E}. : 0 \to \mathcal{E}_1 \to \mathcal{E}_2 \to \mathcal{E}_3 \to 0$ be a short exact sequence of holomorphic vector sheaves on X. We may choose arbitary hermitian metrics ρ_j's on $\mathcal{E}_j$'s for $j = 1,2,3$. Then we have

$$dd^c\phi_{\rm BC}(\mathcal{E}.,\rho_1,\rho_2,\rho_3) = \phi(\mathcal{E}_2,\rho_2) - \phi(\mathcal{E}_1\oplus\mathcal{E}_3,\rho_1\oplus\rho_3)$$

in $\tilde{\mathcal{A}}(X) := \mathcal{A}(X)/(\mathrm{Im}\partial + \mathrm{Im}\overline{\partial})$.

Axiom 2. (Base Change Rule) For any morphism f,

$$f^*\phi_{BC}(\mathcal{E}.,\rho_1,\rho_2,\rho_3) = \phi_{BC}(f^*\mathcal{E}.,f^*\rho_1,f^*\rho_2,f^*\rho_3).$$

Axiom 3. (Uniqueness Rule) If $(\mathcal{E}_2,\rho_2) = (\mathcal{E}_1\oplus\mathcal{E}_3,\rho_1\oplus\rho_3)$, then

$$\phi_{BC}(\mathcal{E}.,\rho_1,\rho_2,\rho_3) = 0.$$

Now we have the following

Bott-Chern Theorem. There exists a unique differential form $\phi_{BC}(\mathcal{E}.,\rho_1,\rho_2,\rho_3)$ which satisfies the above three axioms.

Sketch of the proof. Here we will use the $\mathbf{P}^1$-deformation technique to prove this result.

Let $\mathbf{P}^1$ be the complex projective line. First we will construct an exact sequence $D\mathcal{E}.$ on $X \times \mathbf{P}^1$, called a $\mathbf{P}^1$ deformation of $\mathcal{E}.$.

Let s be a section of $\mathcal{O}_{\mathbf{P}^1}(1)$, which vanishes at ∞ and such that $s(0) = 1$. Let

$$\mathcal{E}_1(1) = \mathcal{E}_1 \otimes \mathcal{O}_{\mathbf{P}^1}(1), \ \tilde{\mathcal{E}}_2 = (\mathcal{E}_2 \oplus \mathcal{E}_1(1))/\mathcal{E}_1$$

with $\mathrm{id}_{\mathcal{E}_1} \otimes s : \mathcal{E}_1 \to \mathcal{E}_1(1)$. Then we have

$$D\mathcal{E}. : \qquad 0 \to \mathcal{E}_1(1) \to \tilde{\mathcal{E}}_2 \to \mathcal{E}_3 \to 0.$$

Now for any point $z \in \mathbf{P}^1$, let $i_z : X \to X \times \mathbf{P}^1$ by $i_z(x) = (x, z)$. We have

1. $\mathcal{E}_2 \simeq i_z^* \tilde{\mathcal{E}}_2$, if $z \neq \infty$.
2. $\mathcal{E}_1 \simeq i_\infty^* \mathcal{E}_1(1)$.
3. $i_\infty^* \tilde{\mathcal{E}}_2 \simeq \mathcal{E}_1 \oplus \mathcal{E}_2$.

Using a partition of unity, we can choose a hermitian metric $\tilde{\rho}_2$ on $\tilde{\mathcal{E}}_2$ in such a way that the isomorphism 1 and 3 above become isometries. Thus we may let

$$\phi_{\mathrm{BC}}(\mathcal{E}., \rho_1, \rho_2, \rho_3) = \int_{\mathbf{P}^1} \log|z|^2 \phi(\tilde{\mathcal{E}}_2, \tilde{\rho}_2).$$

Since

$$d_z d_z^c[\log|z|^2] = \delta_0 - \delta_\infty,$$

we can easily have Axiom 1 by fact 2. Here δ denotes the Dirac distribution. For others, please see Theorem 1.29 of [BGS 88].

§I.2. Relative Bott-Chern Secondary Characteristic Forms For Smooth Morphism

In this section, we state the basic facts associated with the relative Bott-Chern secondary characteristic forms for smooth morphism between complex Kähler manifolds. These relative Bott-Chern secondary characteristic forms may be throught as a natural generalization of the classical Bott-Chern secondary characteristic forms with respect to Chern forms. References here are [BGS 91], [BGV 91] and [W 91a].

Let $f : X \to Y$ be a smooth morphism, which is a Kähler fibration, of compact complex Kähler manifolds with ρ_f the hermitian metric on the relative tangent sheaf $\mathcal{T}_{X/Y}$. Let $(\mathcal{E}, \rho)$ be an f-acyclic hermitian vector sheaf on X, i.e. higher direct images of $\mathcal{E}$ with respect to f vanish. We may introduce a relative version of classical Bott-Chern secondary characteristic form for them by the following axioms. Usually, we will denote the **relative Bott-Chern secondary characteristic form** on Y as $\mathrm{ch}_{\mathrm{BC}}(\mathcal{E}, \rho, f, \rho_f)$.

Axiom 1. (Downstairs Rule)

$$d_Y d_Y^c \mathrm{ch}_{\mathrm{BC}}(\mathcal{E}, \rho, f, \rho_f) = f_*(\mathrm{ch}(\mathcal{E}, \rho)\mathrm{td}(\mathcal{T}_{X/Y}, \rho_f)) - \mathrm{ch}(f_*\mathcal{E}, f_*\rho).$$

Axiom 2. (Base Change Rule) For any base change g, we have

$$g^*\mathrm{ch}_{\mathrm{BC}}(\mathcal{E}, \rho, f, \rho_f) = \mathrm{ch}_{\mathrm{BC}}(g_f^*\mathcal{E}, g_f^*\rho, f_g, g_f^*\rho_f).$$

Here g_f denotes the induced morphism of g with respect to f, and f_g denotes the induced morphism of f with respect to g.

Axiom 3. (Triangle Rule For Hermitian Vector Sheaves) For any short exact sequence of f-acyclic vector sheaves

$$\mathcal{E}.: \qquad 0 \to \mathcal{E}_1 \to \mathcal{E}_2 \to \mathcal{E}_3 \to 0,$$

with hermitian metrics ρ_i on $\mathcal{E}_i$ for $i = 1, 2, 3$, let its direct image be

$$f_*\mathcal{E}.: \qquad 0 \to f_*\mathcal{E}_1 \to f_*\mathcal{E}_2 \to f_*\mathcal{E}_3 \to 0$$

with associated metrics $f_*\rho_i$. We have

$$\mathrm{ch}_{\mathrm{BC}}(\mathcal{E}_2, \rho_2, f, \rho_f) - \mathrm{ch}_{\mathrm{BC}}(\mathcal{E}_1, \rho_1, f, \rho_f) - \mathrm{ch}_{\mathrm{BC}}(\mathcal{E}_3, \rho_3, f, \rho_f) = f_*(\mathrm{ch}_{\mathrm{BC}}(\mathcal{E}., \rho)\mathrm{td}(\mathcal{T}_{X/Y}, \rho_f)) - \mathrm{ch}_{\mathrm{BC}}(f_*\mathcal{E}., f_*\rho).$$

Axiom 4. (Triangle Rule For Morphisms) Let $f : X \to Y$ and $g : Y \to Z$ be two smooth morphisms of complex compact manifolds. Let $(\mathcal{E}, \rho)$ be an f-acyclic hermitian vector sheaf on X such that $f_*\mathcal{E}$ is g-acyclic. We have the short exact sequence for relative tangent sheaves:

$$0 \to \mathcal{T}_{X/Y} \to \mathcal{T}_{X/Z} \to f^*\mathcal{T}_{Y/Z} \to 0,$$

with hermitian metrics ρ_f, $\rho_{g\circ f}$ and ρ_g on $\mathcal{T}_{X/Y}$, $\mathcal{T}_{X/Z}$ and $\mathcal{T}_{Y/Z}$ respectively. Then

$$\begin{aligned}\mathrm{ch}_{\mathrm{BC}}(\mathcal{E}, \rho, g\circ f, \rho_{g\circ f}) \ - \ \mathrm{ch}_{\mathrm{BC}}(f_*\mathcal{E}, f_*\rho, g, \rho_g) \ - \ g_*(\mathrm{ch}_{\mathrm{BC}}(\mathcal{E}, \rho, f, \rho_f)\mathrm{td}(\mathcal{T}_{Y/Z}, \rho_g)) \\ = (g\circ f)_*(\mathrm{ch}(\mathcal{E}, \rho)\mathrm{td}_{\mathrm{BC}}(X, Y, Z)).\end{aligned}$$

Here $\mathrm{td}_{\mathrm{BC}}(X, Y, Z)$ is the Bott-Chern secondary characteristic form associated to the above short exact sequence of the relative hermitian tangent sheaves.

The main result associated with relative Bott-Chern secondary characteristic forms for smooth morphism is the following

Existence Theorem Of Relative Bott-Chern Secondary Characteristic Forms For Smooth Morphism. Let $f : X \to Y$ be a smooth morphism, which is a Kähler fibration, of compact complex Kähler manifolds with ρ_f the hermitian metric on the relative tangent sheaf $\mathcal{T}_{X/Y}$. Let $(\mathcal{E}, \rho)$ be an f-acyclic hermitian vector sheaf on X. Then there exists a unique differential form on Y, $\mathrm{ch}_{\mathrm{BC}}(\mathcal{E}, \rho, f, \rho_f)$, such that the above axioms 1, 2, 3, and 4 hold.

Idea of the proof: The basic idea here is that we first construct a differential form on Y. Then we check that our form satisfies the above axioms.

To construct the relative Bott-Chern secondary characteristic forms, we may imitate the process in the last section for the classical Bott-Chern secondary characteristic forms as follows:

In the last section, we construct the classical Bott-Chern secondary characteristic forms as an integration of certain forms with respect to $\mathbf{P}^1$. Note that if we only want to consider the Bott-Chern secondary vharacteristic forms with respect to Chern forms, we may let ϕ as the exponential function. We do use the same approach to construct our relative Bott-Chern secondary characterictic forms here. For doing this, we have two problems. The first one is that how we can find a canonical connection. The second one is what is our corresponding concept for the exponential function. For the first, it is well-known that we can use Bismut super-connection. For the second, by the works of mathematicians working on index theorem, we know that the corresponding concept for this exponential function is nothing but the associated heat kernel. Once we find these, the construction is the same as in the classical situation.

After we finish the above construction, we can check the our axioms step by step. For example, to check axiom 1, we have to use Bismut local index theorem [B 86] and a result of Berline and Vergne [BGV 91]. More details may be found in [GS 91a], [W 91a].

§I.3. Relative Bott-Chern Secondary Characteristic Current For Closed Immersion

In this section, we recall the theory for relative Bott-Chern secondary characteristic currents for closed immersion. Readers may find the results of this section from [BGS 91].

Let $i : X \hookrightarrow Y$ be a closed immersion of complex manifolds. Let $\mathcal{E}$ be a vector sheaf on X. Let $\mathcal{N}_i$ be the normal sheaf of X in Y. Suppose that we have the following resolution of vector sheaves for the direct image of $i_*\mathcal{E}$:

$$0 \to \mathcal{F}_m \to \mathcal{F}_{m-1} \to \dots \to \mathcal{F}_0 \to i_*\mathcal{E} \to 0.$$

For any $x \in X$, let $F_{k,x}$ be the k^{th} homology group of complex

$$0 \to \mathcal{F}_m \to \mathcal{F}_{m-1} \to \dots \to \mathcal{F}_0 \to 0.$$

Set $F_x = \oplus F_{k,x}$. Thus if we metrize the above situation, we may put hermitian metrics on all vector sheaves. Hence we may introduce the formal adjoints for the boundary maps.

By a generalized Hodge theorem, we know that

$$F_k = \{\omega \in \mathcal{F}_k \mid v\omega = 0, v^*\omega = 0\}.$$

Naturally, on F_k, there is an induced metric.

By the local uniqueness of resolutions, we have the following statements:

1. For $k = 0, 1, ..., m, x \in X$, the dimension of $F_{k,x}$ is constant on each irreducible component of X. So that F_k is a holomorphic vector sheaf on X.

2. For $x \in X, u \in \mathcal{T}_x X$, let $\partial_u v(x)$ be the derivative of the chain map v calculated in any given local trivialization of $(\mathcal{F}., v.)$ near x. Then $\partial_u v(x)$ acts on F_x. When acting on F_x, $\partial_u v(x)$ only dependents on the image y of u in $\mathcal{N}_{i,x}$. So we will write $\partial_y v(x)$ instead of $\partial_u v(x)$.

3. For any $x \in X, y \in \mathcal{N}_i$, $(\partial_y v)^2(x) = 0$. If $y \in \mathcal{N}_i$, let i_y be the interior multiplication operator by y acting on the exterior algebra $\wedge\mathcal{N}^*$. Let i_y act like $i_y \otimes 1$ on $\wedge\mathcal{N}^* \otimes \mathcal{E}$. Then the graded holomorphic complex $(F., \partial_y v.)$ on the total space of the vector sheaf $\mathcal{N}_i$ is canonically isomorphic to the Koszul complex $(\wedge\mathcal{N}_i^* \otimes \mathcal{E}, i_y)$. Furthermore, for any given metric $\rho_{\mathcal{N}_i}, \rho_{\mathcal{E}}$ on $\mathcal{N}_i, \mathcal{E}$ respectively, there always exist metrics ρ_k on $\mathcal{F}_k$ such that the above canonical algebraic isomorphism becomes an isometry (Usually, we will call the condition here as **Bismut condition (A)**).

Next we will introduce the concept: **wave front**.

If γ is a current on Y, we denote $\mathrm{WF}(\gamma)$ the wave front set of γ. For the definition and the properties of the wave front set, please see [Hö 83]. Especially, we know that $\mathrm{WF}(\gamma)$ is a closed conic subset of $\mathcal{T}_{\mathbf{R}}^* Y - \{0\}$. Also if p is the projection from the total space of cotangent sheaf of Y to Y, $p\mathrm{WF}(\gamma)$ is exactly the singular support of γ, whose complement in Y is the set of points x such that γ is C^∞ on a neighborhood of x. Usually, we denote $\mathcal{D}_{\mathcal{N}_{\mathbf{R}}^*}$ the set of currents γ on Y which satisfy that $\mathrm{WF}(\gamma) \subset \mathcal{N}_{\mathbf{R}}^*$. Thus the elements in $\mathcal{D}_{\mathcal{N}_{\mathbf{R}}^*}$ are smooth on $Y - X$. Also there is a natural topology on $\mathcal{D}_{\mathcal{N}_{\mathbf{R}}^*}$ which may be defined as follows:

Let U be a small open set in Y, which we identify with an open ball in $\mathbf{R}^{2l}$. Over U, we identify $T^*_{\mathbf{R}}Y$ with $U \times \mathbf{R}^{2l}$. Let Γ be a closed conic set in $\mathbf{R}^{2l}$ such that if $x \in U$, $\Gamma \cap \mathcal{N}^*_{\mathbf{R}x} = \emptyset$. Let φ be a smooth current on $\mathbf{R}^{2l}$ with compact support included in U and let m be an integer. If γ is a current, let $\hat{\varphi\gamma}$ be the Fourier transform of $\varphi\gamma$ (which is considered as a current on $\mathbf{R}^{2l}$ here). If $\gamma \in \mathcal{D}_{\mathcal{N}^*_{\mathbf{R}}}$, set

$$p_{u,\Gamma,\varphi,m}(\gamma) := \sup_{\xi \in \Gamma} |\xi|^m |\hat{\varphi\gamma}(\xi)|.$$

With this, we say a sequence of current $\{\gamma_n\}$ in $\mathcal{D}_{\mathcal{N}^*_{\mathbf{R}}}$ converges to $\gamma \in \mathcal{D}_{\mathcal{N}^*_{\mathbf{R}}}$, if

1. $\lim_{n\to\infty}\gamma_n = \gamma$ in the sense of distributions.

2. $\lim_{n\to\infty} p_{u,\Gamma,\varphi,m}(\gamma_n - \gamma) = 0$.

Usually, we also let

$$P^Y_X := \{\omega \in \mathcal{D}_{\mathcal{N}^*_{\mathbf{R}}} \mid \omega \text{ is a sum of currents of type (p, p)}\}.$$

And

$$P^{Y,0}_X := \{\omega \in P^Y_X \mid \omega = \partial\alpha + \bar{\partial}\beta \text{ with } \alpha,\ \beta \in \mathcal{D}_{\mathcal{N}^*_{\mathbf{R}}}\}.$$

If $X = \emptyset$, we also wirte $P^Y, P^{Y,0}$ instead of $P^Y_X, P^{Y,0}_X$ respectively.

With these notations, we may introduce the axioms for **relative Bott-Chern secondary characteristic current** for closed immersion, $\mathrm{ch}_{\mathrm{BC}}(\mathcal{E}, \rho, i, \rho_i)$, as follows:

Axiom 1. (Transgression Formula)

$$dd^c \mathrm{ch}_{\mathrm{BC}}(\mathcal{E}, \rho, i, \rho_i) = \mathrm{td}^{-1}(\mathcal{N}, g_{\mathcal{N}})\mathrm{ch}(\mathcal{E}, \rho)\delta_X - \mathrm{ch}(\mathcal{E}, \rho).$$

Axiom 2. (Base Change Rule) Let $f : \tilde{Y} \to Y$ be a holomorphic morphism. Assume that f is transversal to $\tilde{Y}$. That is, for any $x \in f^{-1}(X)$,

$$\mathrm{Im}\ df(x)\ +\ T_{f(x)}X = T_x Y.$$

Then we have

$$\mathrm{ch}_{\mathrm{BC}}(f^*\mathcal{E}, f^*\rho, i_f, \rho_{i_f}) = f^*\mathrm{ch}_{\mathrm{BC}}(\mathcal{E}, \rho, i, \rho_i).$$

Axiom 3. (Triangle Rule For Hermitian Vector Sheaves) Let

$$0 \to \mathcal{E}_1 \to \mathcal{E}_2 \to \mathcal{E}_3 \to 0$$

be a short exact sequence of vector sheaves on X. Then we may find the resolution $\mathcal{F}_{k,\cdot}$ for $i_*\mathcal{E}_k$ in the above sense, with the condition that

$$0 \to \mathcal{F}_{1,j} \to \mathcal{F}_{2,j} \to \mathcal{F}_{3,j} \to 0$$

is a short exact sequence. Put metrics satisfying Bismut condition (A) on them. Then we have

$$\sum_{k=1}^{3}(-1)^k \mathrm{ch}_{\mathrm{BC}}(\mathcal{E}_k, \rho_k, i, \rho_i) =$$

$$i_*(\mathrm{td}^{-1}(\mathcal{N}, \rho_{\mathcal{N}})\mathrm{ch}_{\mathrm{BC}}(\mathcal{E}, \rho, i, \rho_i)) - \sum_{j=0}^{m}(-1)^j \mathrm{ch}_{\mathrm{BC}}(\mathcal{F}_{\cdot,j}, \rho_{\cdot,j})$$

in $P_X^Y / P_X^{Y,0}$.

Axiom 4. (Triangle Rule For Closed Immersions) Let $i' : X' \hookrightarrow Y$ be another closed immersion such that X and X' intersect transversally, i.e. if $x \in X \cap X'$, then

$$\mathcal{T}_x X + \mathcal{T}_x X' = \mathcal{T}_x Y.$$

Let $i'' : X'' = X \cap X' \hookrightarrow Y$ be the induced closed immersion. For any vector sheaves $\mathcal{E}$ (resp. $\mathcal{E}'$) on X (resp. X'), let

$$\mathcal{E}'' := \mathcal{E}|_X \otimes \mathcal{E}'|_{X'}.$$

Then in $P^Y_{X \cup X'} / P^{Y,0}_{X \cup X'}$, we have

$$\mathrm{ch}_{\mathrm{BC}}(\mathcal{E}'', \rho'', i'', \rho_{i''}) =$$

$$= \mathrm{ch}(\mathcal{F}'_\cdot, \rho'_\cdot)\mathrm{ch}_{\mathrm{BC}}(\mathcal{E}, \rho, i, \rho_i) + i_*(\mathrm{td}^{-1}(\mathcal{N}, \rho_{\mathcal{N}})\mathrm{ch}(\mathcal{E}, \rho_{\mathcal{E}}) i^* \mathrm{ch}_{\mathrm{BC}}(\mathcal{E}', \rho', i', \rho_{i'})$$

and

$$\mathrm{ch}_{\mathrm{BC}}(\mathcal{E}'', \rho'', i'', \rho_{i''}) =$$

$$\mathrm{ch}(\mathcal{F}_., \rho_.)\mathrm{ch}_{\mathrm{BC}}(\mathcal{E}', \rho', i', \rho_{i'}) + i'_*(\mathrm{td}^{-1}(\mathcal{N}', \rho_{\mathcal{N}'})\mathrm{ch}(\mathcal{E}', \rho_{\mathcal{E}'})i'^*\mathrm{ch}_{\mathrm{BC}}(\mathcal{E}, \rho, i, \rho_i).$$

With this, we may state the main results of [BGS 91] as the following

Existence Theorem Of Relative Bott-Chern Secondary Characteristic Current For Closed Immersion. With the notation as above, let $i : X \to Y$ be a closed immersion of compact complex Kähler manifolds. Let $(\mathcal{E}, \rho)$ be a hermitian vector sheaf on X. Then there exists a unique current in P_X^Y, $\mathrm{ch}_{\mathrm{BC}}(\mathcal{E}, \rho, f, \rho_f)$, such that the above axioms 1, 2, 3, and 4 hold.

The proof of this existence theorem may be found in [BGS 91].

Part II. Arithmetic Riemann-Theorem For l.c.i. Morphism

In this part, we will use the results in the first part to give the arithmetic Riemann-Roch theorem for l.c.i. morphism by pure algebraic methods.

§II.1. Push-Out Morphism For Arithmetic K-Groups

In this section we will give a definition of push-out morphism of arithmetic K-groups for l.c.i. morphism of regular arithmetic varieties, but with a technical assumption that f at infinity is smooth.

Let $f : X \to Y$ be a l.c.i. morphism between regular arithmetic varieties. It is well-known that for l.c.i. morphism f, we have the following decompsition: a closed immersion $i : X \hookrightarrow P$ followed by a projection $g : P \to Y$. Usually, we may try to define

$$f_K : \mathrm{K}_{\mathrm{Ar}}(X)_{\mathbf{Q}} \to \mathrm{K}_{\mathrm{Ar}}(Y)_{\mathbf{Q}}$$

by introducing the push-out morphism of arithmetic K-groups for both smooth morphism and closed immersion. For a smooth morphism, there is no problem (see below). For closed immersion, we have a problem: It can be done in principle, but the problem now is that we need a systematic theory for relative arithmetic intersection theory and relative arithmetic K-theory for closed immersion. Since we do not want to develop the relative theory here, we will give a direct definition of push-out morphism of arithmetic K-groups for l.c.i. morphism.

Note that since the arithmetic K-group $\mathrm{K}_{\mathrm{Ar}}(X)$ is generated by f-acyclic hermitian vector sheaves and smooth forms, we only need to give the definition of f_K for both of these kinds of elements, and to prove the compatibility of our definition.

For doing this, let us introduce a few more notations. For the above decompsition of f, since $f_{\mathbf{C}}$ is smooth, we may have the following short exact sequence:

$$N : 0 \to \mathcal{T}_{f_{\mathbf{C}}} \to i^*\mathcal{T}_{g_{\mathbf{C}}} \to \mathcal{N}_{i,\mathbf{C}} \to 0.$$

With this, for any $\Upsilon \in \mathrm{CH}_{\mathrm{Ar}}(X)_{\mathbf{Q}}$, as in [GS 91b], we define

$$\mathrm{td}_{\mathrm{Ar}}(f, \rho_f)\Upsilon :=$$
$$= \mathrm{td}_{\mathrm{Ar}}(i^*\mathcal{T}_g, i^*\rho_g)(\mathrm{td}_{\mathrm{Ar}}{}^{-1}(\mathcal{N}_i, \rho_{\mathcal{N}_i})\Upsilon) + \mathrm{td}_{\mathrm{BC}}(f/g, \rho_{f/g})\Upsilon \in \mathrm{CH}_{\mathrm{Ar}}(X)_{\mathbf{Q}}.$$

Here $\mathrm{td}_{\mathrm{BC}}(f/g, \rho_{f/g})$ is the intersection of the classical Bott-Chern secondary characteristic forms associated with the short exact sequence N above and $\mathrm{td}_{\mathrm{Ar}}^{-1}(\mathcal{N}_i, \rho_{\mathcal{N}_i})$. And as usual we omit the notation for the morphism a, which send a smooth form α to an arithmetic cycle $(0, \alpha)$.

Now, for smooth forms, there is no problem, as the arithmetic Riemann-Roch theorem is supposed to be a generalization of the classical Riemann-Roch theorem. Therefore, if $\alpha \in \mathrm{K}_{\mathrm{Ar}}(X)$, it is natural for us to let

$$f_K(\alpha) := f_*(\alpha\ \mathrm{td}_{\mathrm{Ar}}(f, \rho_f)).$$

Next we will give the definition of f_K for f-acyclic hermitian vector sheaves. Let $(\mathcal{E}, \rho)$ be an f-acyclic hermitian vector sheaf on X. From part I, we know that there is a resolution of g-acyclic vector sheaves on P for $i_*\mathcal{E}$:

$$0 \to \mathcal{F}_n \to \dots \to \mathcal{F}_1 \to \mathcal{F}_0 \to i_*\mathcal{E} \to 0.$$

Equiped $\mathcal{F}_j$ with metrics ρ_j, which satisfy Bismut condition (A), we may give the following definition:

$$\begin{aligned} f_K^P(\mathcal{E},\rho) =: & \\ &= \sum_{j=0}^{n}(-1)^j(g_*\mathcal{F}_j, g_*\rho_j) + \sum_{j=0}^{n}(-1)^j \mathrm{ch}_{\mathrm{BC}}(\mathcal{F}_j, \rho_j, g, \rho_g) \\ &+ \sum_{j=0}^{n} g_*(\mathrm{ch}_{\mathrm{Ar}}(\mathcal{F}_j, \rho_j)\mathrm{td}_{\mathrm{Ar}}(g, \rho_g)P(g, \rho_g)) + g_*(\mathrm{ch}_{\mathrm{BC}}(\mathcal{E}, \rho, i, \rho_i)\mathrm{td}_{\mathrm{Ar}}(g, \rho_g)) \\ &+ f_*(\mathrm{ch}_{\mathrm{Ar}}(\mathcal{E}, \rho)\mathrm{td}_{\mathrm{BC}}(f/g, \rho_{f/g})). \end{aligned}$$

Here we use the isomorphism $\mathrm{ch}_{\mathrm{Ar}}$ to think the elements

$$\sum_{j=0}^{n} g_*(\mathrm{ch}_{\mathrm{Ar}}(\mathcal{F}_j, \rho_j)\mathrm{td}_{\mathrm{Ar}}(g, \rho_g)P(g, \rho_g))$$

and

$$g_*(\mathrm{ch}_{\mathrm{BC}}(\mathcal{E}, \rho, i, \rho_i)\mathrm{td}_{\mathrm{Ar}}(g, \rho_g)) + f_*(\mathrm{ch}_{\mathrm{Ar}}(\mathcal{E}, \rho)\mathrm{td}_{\mathrm{BC}}(f/g, \rho_{f/g}))$$

in $\mathrm{CH}_{\mathrm{Ar}\mathbf{Q}}$ as those in $K_{\mathrm{Ar}\mathbf{Q}}$. Note that since they are in the image of a, the mean of those elements in $K_{\mathrm{Ar}\mathbf{Q}}$ are the same as those in $\mathrm{CH}_{\mathrm{Ar}\mathbf{Q}}$. The P term above corresponds to the associated additive homology class for any power series $P(x) = \sum_{k\geq 0} a_k x^k$, which is defined as follows: for any hermitian line sheaf $(\mathcal{L}, \rho)$

$$P(\mathcal{L}, \rho) := \sum_{k\geq 0} a_k c_1(\mathcal{L}, \rho)^k.$$

Now we have the following

Proposition. With the above definition for smooth forms and f-acyclic hermitian sheaves, we have a well-defined group morphism

$$f_K^P : \mathrm{CH}_{\mathrm{Ar}}(X)_{\mathbf{Q}} \to \mathrm{CH}_{\mathrm{Ar}}(Y)_{\mathbf{Q}}.$$

Here one may ask the dependence of f_K^P on the various data. But this is a direct consequence of the following main result of this paper

Arithmetic Riemann-Roch Theorem For l.c.i. Morphism. There exists a unique power series $P(x)$ such that, for any l.c.i. morphism $f : X \to Y$ of regular arithmetic varieties with f at infinity is smooth, and for any element $\Upsilon \in \mathrm{CH}_{\mathrm{Ar}}(X)_{\mathbf{Q}}$, we have

$$\mathrm{ch}_{\mathrm{Ar}} f_K^P(\Upsilon) := f_{\mathrm{CH}}(\Upsilon\ \mathrm{td}_{\mathrm{Ar}}(f, \rho_f)).$$

Usually, we denote f_K^P as f_K.

In fact, by proposition 1 of 2.6.2 of [GS 91b], we know that $\mathrm{td}_{\mathrm{Ar}}(f, \rho_f)$ depends only on the choice of metric on $\mathcal{T}_{f_{\mathbf{C}}}$, and not on the choice of i, g, nor on the metrics on $\mathcal{N}_i$ and $\mathcal{T}_g$. Therefore, we know that our f_K also depends only on the choice of the metric on $\mathcal{T}_{f_{\mathbf{C}}}$, and not on others.

§II.2. Sketch Of The Proof Of Arithmetic Riemann-Roch Theorem

We divide the proof of arithmetic Riemann-Roch theorem for l.c.i. morphism into two steps.

First, we have to verify our result for smooth morphisms, which has its root in [F 91]. Then we prove the arithmetic Riemann-Roch theorem in general by combining the cases for both smooth morphism and closed immersion.

For smooth morphism f, we can assume that i is an identity morphism and $g = f$ is a smooth morphism. Note that since the definition in the last section make sense in this situation, we may define

$$\mathrm{Err}(\mathcal{E}, \rho; f, \rho_f; P) := \mathrm{ch}_{\mathrm{Ar}} f_K^P(\mathcal{E}, \rho) - f_{\mathrm{CH}}(\mathrm{ch}_{\mathrm{Ar}}(\mathcal{E}, \rho)\ \mathrm{td}_{\mathrm{Ar}}(f, \rho_f)).$$

By the existence theorem of the relative Bott-Chern secondary characteristic forms for smooth morphisms, we can show that $\mathrm{Err}(\mathcal{E},\rho;f,\rho_f;P)$ does not depend on the metrics and in fact defines a morphism from $K(X_{\mathbf{Q}})$ to the image of a. (For this reason, we usually let

$$\mathrm{Err}(\mathcal{E},f,P):=\mathrm{Err}(\mathcal{E},\rho;f,\rho_f;P).)$$

Now we have to prove Err is 0 for a fixed power series $P(x)$. For doing this, we may also introduce an error therm for a closed immersion. That is,

$$\mathrm{Err}(\mathcal{E},i,P):=\mathrm{Err}(\mathcal{E},f,P)-\mathrm{Err}(i_*\mathcal{E},g,P).$$

Now we may prove that if Err is 0 for both $\mathbf{P}^1$-bundles and codimension one closed immersions, then Err is 0 in general, by taking certain standard results from algebraic K-theory. But for $\mathbf{P}^1$-bundles, as the algebraic K-groups has a relatively simple structure, we can check directly. On the other hand, for codimensional one closed immersions, we have to use a result for relative Bott-Chern secondary characrteristic forms with respect to the theory of deformation to the normal cone. (In fact, this relation between relative Bott-Chern secondary characteristic forms and the theory of deformation to the normal cone will finally give us the so-called Bott-Chern ternary objects. As there is no obvious application for them, we do not study them here.) More details may be found in [F 91] and [W 91a].

Now we give the proof of arithmetic Riemann-Roch theorem in general. This is a direct consequence of the arithmetic Riemann-Roch theorem for smooth morphism and the following result of [BGS 91]:

Arithmetic Riemann-Roch Theorem For Closed Immersion With the notation as above, let $i:X\to P$ be a closed immersion of regular arithmetic varieties over a regular arithmetic variety Y with smooth srtucture morphisms $f:X\to Y$ and $g:P\to Y$. Let $\mathcal{E}$ be an f-acyclic hermitian vector sheaf on X. Then we have: for any Υ in $\mathrm{CH}_{\mathrm{Ar}}(P)$,

$$g_{\mathrm{CH}}(\mathrm{ch}_{\mathrm{Ar}}(\mathcal{F}.,\rho.)\,\Upsilon)=f_{\mathrm{CH}}(\mathrm{ch}_{\mathrm{Ar}}(\mathcal{E},\rho)\mathrm{td}_{\mathrm{Ar}}(\mathcal{N},\rho_{\mathcal{N}})^{-1}i^*\Upsilon)-g_{\mathrm{CH}}(\mathrm{ch}_{\mathrm{BC}}(\mathcal{E},\rho,i,\rho_i)\omega(\Upsilon)).$$

Here ω denotes the quasi-forgetting morphism from $\mathrm{CH}_{\mathrm{Ar}}$ to $A(X(\mathbf{C}))$:

$$\omega(Z, g_Z) = dd^c g_Z + \delta_z.$$

Remark: The condition of that f is smooth is not necessary for certain purpose, say taking α as $\mathrm{td}_{\mathrm{Ar}}(g, \rho_g)$. We only need to assume that f is a l.c.i. morphism with that f at infinity is smooth: This can be achieved by operation formalism of Fulton [FL 85]. For more details, please see 4.2.3 of [GS 91b].

Proof of arithmetic Riemann-Roch theorem for l.c.i. morphism. Obviously, it is sufficient for us to prove the formula for f-acyclic hermitian vector sheaves. Let $(\mathcal{E}, \rho)$ be such an element, we have

$$\begin{aligned}
&\mathrm{ch}_{\mathrm{Ar}} f_K(\mathcal{E},\rho) - f_{\mathrm{CH}}(\mathrm{ch}_{\mathrm{Ar}}(\mathcal{E},\rho)\ \mathrm{td}_{\mathrm{Ar}}(f,\rho_f))\\
&\quad= \sum_{j=0}^{n}(-1)^j \mathrm{ch}_{\mathrm{Ar}}(g_*\mathcal{F}_j, g_*\rho_j) + \sum_{j=0}^{n}(-1)^j \mathrm{ch}_{\mathrm{BC}}(\mathcal{F}_j, \rho_j, g, \rho_g)\\
&\quad+ \sum_{j=0}^{n} g_*(\mathrm{ch}_{\mathrm{Ar}}(\mathcal{F}_j,\rho_j)\mathrm{td}_{\mathrm{Ar}}(g,\rho_g)P(g,\rho_g)) + g_*(\mathrm{ch}_{\mathrm{BC}}(\mathcal{E},\rho,i,\rho_i)\mathrm{td}_{\mathrm{Ar}}(g,\rho_g))\\
&\quad+ f_*(\mathrm{ch}_{\mathrm{Ar}}(\mathcal{E},\rho)\mathrm{td}_{\mathrm{BC}}(f/g,\rho_{f/g})) - f_{\mathrm{CH}}(\mathrm{ch}_{\mathrm{Ar}}(\mathcal{E},\rho)\ \mathrm{td}_{\mathrm{Ar}}(f,\rho_f))\\
&\quad= \sum_{j=0}^{n}(-1)^j \mathrm{ch}_{\mathrm{Ar}}(g_K(\mathcal{F}_j,\rho_j)) + g_*(\mathrm{ch}_{\mathrm{BC}}(\mathcal{E},\rho,i,\rho_i)\mathrm{td}_{\mathrm{Ar}}(g,\rho_g))\\
&\quad+ f_*(\mathrm{ch}_{\mathrm{Ar}}(\mathcal{E},\rho)\mathrm{td}_{\mathrm{BC}}(f/g,\rho_{f/g})) - f_{\mathrm{CH}}(\mathrm{ch}_{\mathrm{Ar}}(\mathcal{E},\rho)\ \mathrm{td}_{\mathrm{Ar}}(f,\rho_f)).
\end{aligned}$$

By arithmetic Riamenn-Roch theorem for smooth morphism g, we know that the above combination of terms is equal to

$$\begin{aligned}
&\sum_{j=0}^{n}(-1)^j g_{\mathrm{CH}}(\mathrm{ch}_{\mathrm{Ar}}(\mathcal{F}_j,\rho_j)\mathrm{td}_{\mathrm{Ar}}(g,\rho_g)) + g_*(\mathrm{ch}_{\mathrm{BC}}(\mathcal{E},\rho,i,\rho_i)\mathrm{td}_{\mathrm{Ar}}(g,\rho_g))\\
&+ f_*(\mathrm{ch}_{\mathrm{Ar}}(\mathcal{E},\rho)\mathrm{td}_{\mathrm{BC}}(f/g,\rho_{f/g})) - f_{\mathrm{CH}}(\mathrm{ch}_{\mathrm{Ar}}(\mathcal{E},\rho)\ \mathrm{td}_{\mathrm{Ar}}(f,\rho_f)).
\end{aligned}$$

On the other hand, by the arithmetic Riemann-Roch theorem for closed immersion i, we know that the last quantity is

$$\begin{aligned}
&f_*(\mathrm{ch}_{\mathrm{Ar}}(\mathcal{E},\rho){\mathrm{td}_{\mathrm{Ar}}}^{-1}(\mathcal{N}_i,\rho_{\mathcal{N}_i})\ i^*\mathrm{td}_{\mathrm{Ar}}(g,\rho_g) + \mathrm{ch}_{\mathrm{Ar}}(\mathcal{E},\rho)\mathrm{td}_{\mathrm{BC}}(f/g,\rho_{f/g})\\
&- \mathrm{ch}_{\mathrm{Ar}}(\mathcal{E},\rho)\mathrm{td}_{\mathrm{Ar}}(f,\rho_f)),
\end{aligned}$$

which is 0 by definition.

Remark 1. If we look at the classical Kodaira vanishing theorem, we will have certain good feeling to find the analogue here, hence about arithmetic vanishing theorem.

Remark 2. By our arithmetic Riemann-Roch theorem, we can deduce Deligne's Riemann-Roch theorem for semi-stable arithmetic surfaces up to isometry class and also give the uncertain constants $a(g)$, which were firstly offered by Jorgenson [J 91] via his degeneration method, in Deligne's Riemann-Roch theorem.

Remark 3. One may deal with the Riemann-Roch theorem for a general l.c.i. morphism $f : X \to Y$ with $X_{\mathbf{Q}}$ and Y are regular. The first step is to use τ-construction in [GS 91b] instead of $\mathrm{ch}_{\mathrm{Ar}}$. The second step is that we have to give a good definition for arithmetic Todd genus in this situation. But this may be achieved by considering the arithmetic K-group associated with coherent sheaves.

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