Hausdorff dimension of collision times in one-dimensional log-gases

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We consider the interacting particle systems in one dimension described by Dyson's model and the Wishart-Laguerre processes, in which there is a fixed number of particles and the coupling constant of interaction β is less than unity. In this case, it is well-known that collisions occur between the particles almost surely in the sense that the first collision time is finite with probability one, whereas the first collision time is almost surely infinite when β is greater than or equal to one [D]. In spite of this fact, the set of collision times is known to have Lebesgue measure zero [CL], which means that the stochastic differential equations that describe these systems have unique strong solutions in spite of the fact that their drift terms diverge whenever a collision occurs.

A puzzling aspect of the collision times set is that, for the Dyson model of two particles, one can show that collisions occur infinitely often after they happen for the first time [K]. This is because the two-particle case can be reduced to the well-known Bessel process, and the set of collision times of the former corresponds to the returning times to the origin of the latter. It turns out that these characteristics are explained by the fact that the self-similarity enjoyed by the Bessel processes implies that its set of return times to the origin has fractal structure [LX]. In this work, we extend this analysis to an arbitrary and fixed number of particles in both the Dyson model and the Wishart Laguerre processes, and we identify the Hausdorff dimension of collision times as an appropriate object to characterize this behavior by extending the techniques of [LX] to non-compact returning sets.

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