SPHERICAL HARMONICS AND HARDY'S INEQUALITIES

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We consider Hardy's inequality. Its standard form is the following, for $n \geq 3$,

$$\left(\frac{n-2}{2}\right)\left\|\frac{f}{|x|}\right\|_{L^2} \leq \|\nabla f\|_{L^2}$$

where $L^2 = L^2(\mathbb{R}^n)$. It is known that this inequality holds even if we replace the right-hand side by $\|\partial_r f\|_2$ where ∂_r is the differential operator in the radial direction.

In this talk, we decompose the differential operator "nabla ∇" into two roles, the differential operators in radial direction and spherical direction. We consider the roles of them through investigations of Hardy's inequality. We derive some equalities which are expressed by the orthogonal projection to a class of spherically symmetric functions, and more, by the orthogonal projection to the spaces of solid spherical harmonics of degree each $k = 1, 2, \ldots$.

This work is based on a joint research with Bez Neal and Tohru Ozawa.