Hyperuniformity of the determinantal point processes associated with the Heisenberg group

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The *Ginibre point process* is given by the eigenvalue distribution of a non-hermitian complex Gaussian matrix in the infinite matrix-size limit. This is a *determinantal point process* (DPP) on the complex plane $\mathbb{C}$ in the sense that all correlation functions are given by determinants specified by an integral kernel called the *correlation kernel* [1]. As extensions of the Ginibre DPP, a one-parameter family of DPPs is defined on $\mathbb{C}^d$, in which $d \in \mathbb{N}$ is the parameter and the Ginibre DPP is regarded as the lowest-dimensional case ($d = 1$). We call it the *Heisenberg family of DPPs*, and we will explain this reason in the first part of the present talk: For each $d \in \mathbb{N}$, the correlation kernel is expressed by a matrix-element function of the *Schrödinger representation* of the *Heisenberg group*, and is identified with the *reproducing kernel* of the *Bargmann-Fock space*. Then we prove that any DPP in this family has *hyperunifomity* as follows [2]: We consider a series of bounded domains $\Lambda_n \subset \mathbb{C}^d$, which are monotonically increasing and $\Lambda_n \to \mathbb{C}^d$ as $n \to \infty$. They provide a series of 'observation windows' to measure local density-fluctuation for an infinite point process Ξ, and then the hyperuniformity is defined by

$$\lim_{n\to\infty} \frac{\mathrm{var}[\Xi(\Lambda_n)]}{\mathbf{E}[\Xi(\Lambda_n)]} = 0.$$

We show that when Λ_n are polydisks, the proof is readily given by using the *duality relation* between DPPs [3]. In the case that Λ_n are balls, we have derived exact formulas of $\mathrm{var}[\Xi(\Lambda_n)]$ using the modified Bessel functions, which provide asymptotic expansions for $\mathrm{var}[\Xi(\Lambda_n)]$ in $\mathrm{vol}[\Lambda_n] \to \infty$. This talk is based on the joint work with T. Matsui (Chuo Univ.) and T. Shirai (Kyushu Univ.), and with S. Koshida (Aalto Univ.).

References

[1] M. Katori, *Bessel Processes, Schramm–Loewner Evolution, and the Dyson Model*, Springer Briefs in Mathematical Physics 11 (Springer, Singapore, 2016); pdf file is free from the Springer Link, Mathematics and Statistics eBook Collection.

[2] T. Matsui, M. Katori and T. Shirai, Local number variances and hyperuniformity of the Heisenberg family of determinantal point processes, *J. Phys. A: Math. Theor.* **54** (2021) 165201, 22pp.

[3] M. Katori and T. Shirai, Partial isometries, duality, and determinantal point processes, *Random Matrices: Theory and Applications* (2021) 2250025, 70pp; open access `https://www.worldscientific.com/doi/abs/10.1142/S2010326322500253`