

We analyze how phase transitions happen in the BCS model with imaginary magnetic field at positive temperature. We introduce a class of non-vanishing free dispersion relations and characterize the nature of phase transitions in the model with such a dispersion relation. In particular we provide necessary and sufficient conditions for existence of a temperature-driven higher order phase transition in terms of stationary points of inflection on the phase boundary or ratio of minimum value of a free dispersion relation to its maximum. The main object we analyze is the free energy density, which has been derived from the many-electron system in the preceding papers. So our analysis reveals true properties of the infinite-volume limit of the many-Fermion system at positive temperature.